Initial Analyses of Depletion Experiments for Monkfish

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Southern Demersal Working Group

SARC 50

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The Catchability Coefficient
A Fundamental Property in Fisheries Science

- Catchability is a scalar that translates indices of relative abundance to absolute abundance.

- The catchability coefficient q was first defined in 1918 by Baranov who called it the "elemental intensity of fishing" which is the fractional reduction in average density per application of a unit of effort.

- Catchability is a parameter in every fisheries model that combines indices of abundance and estimates of total removals. If the model estimates absolute biomass or numbers, then there is a q buried in the equation sets.

- Index = Catchability x Absolute Abundance

- Model-based estimates of catchability can be a source of instability in dynamic models.

Simple Depletion Models
Leslie and Davis 1939, DeLury 1947

Primary assumptions include

1) All extant individuals have the same probability of being caught in a sample,
2) Expected catch in a sample is proportional to sampling effort,
3) Units of sampling effort are independent and additive,
4) Catch depends on the cumulative catch of preceding samples, and
5) All removals are known.

Assumptions are violated in subtle but important ways

- Variations with size of animal
- Changes in availability to gear
- Changes in behavior of animals
- Loss of animals as a result of sampling

The Basic Depletion Model

Catch = P(Capture|Encounter) x P(Encounter) x Population

$$(1) \qquad \hat{E}(C) = e\left(\frac{a}{A}\right)N$$

Where

e = gear efficiency
a= area swept by unit of gear
A= Total area occupied by population
N = total population size
q = e (a/A)

Note that density D = N/A so that

C = e a D

Deriving the Recursive Model

• Consider a population of size N_o in an area of size A such that the initial density is $D_o = N_o/A$.

Let $C_j = q\, N_{j-1}$ and $N_{j+1} = N_j - C_j$

• Then $C_1 = q\, N_0$ and $N_1 = N_0 - C_1$
• Then $C_2 = q\, N_1$ and $N_2 = N_1 - C_2$
• Then $C_3 = q\, N_2$ and $N_3 = N_2 - C_3$
• Lather, rinse, repeat to get

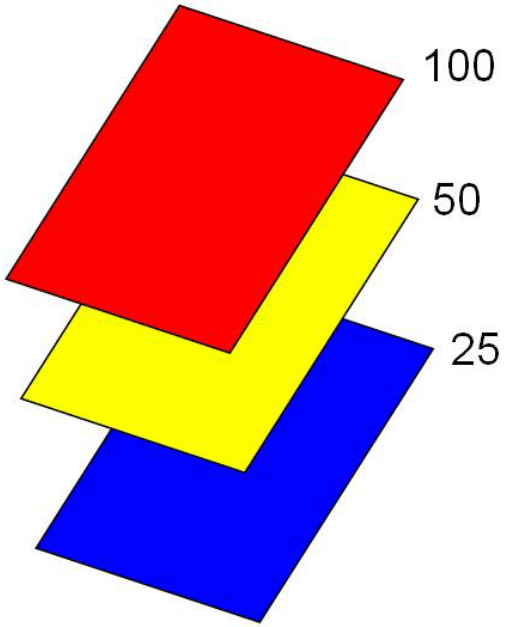
$$(2) \qquad E(C_i) = q\left(N_0 - T_{i-1}\right)$$

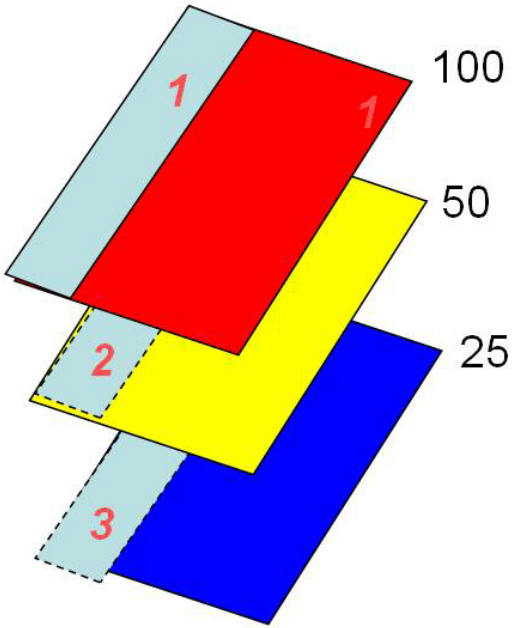
$$\text{where} \quad T_{i-1} = \sum_{j=1}^{i-1} C_j \quad ,$$

$$\text{for} \quad i = \text{tow number}.$$

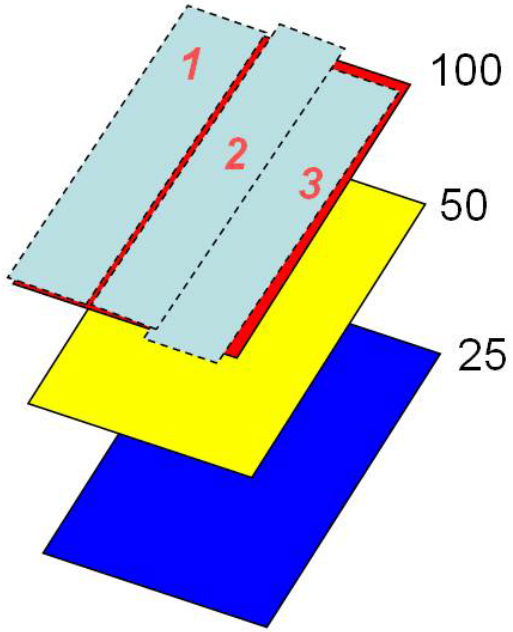
Why Space Matters

- Consider a population that exists over some area A and that the population N is sessile. Assume that the sampling device has a 50% efficiency and that it can sample 1/3 of the area per unit effort.

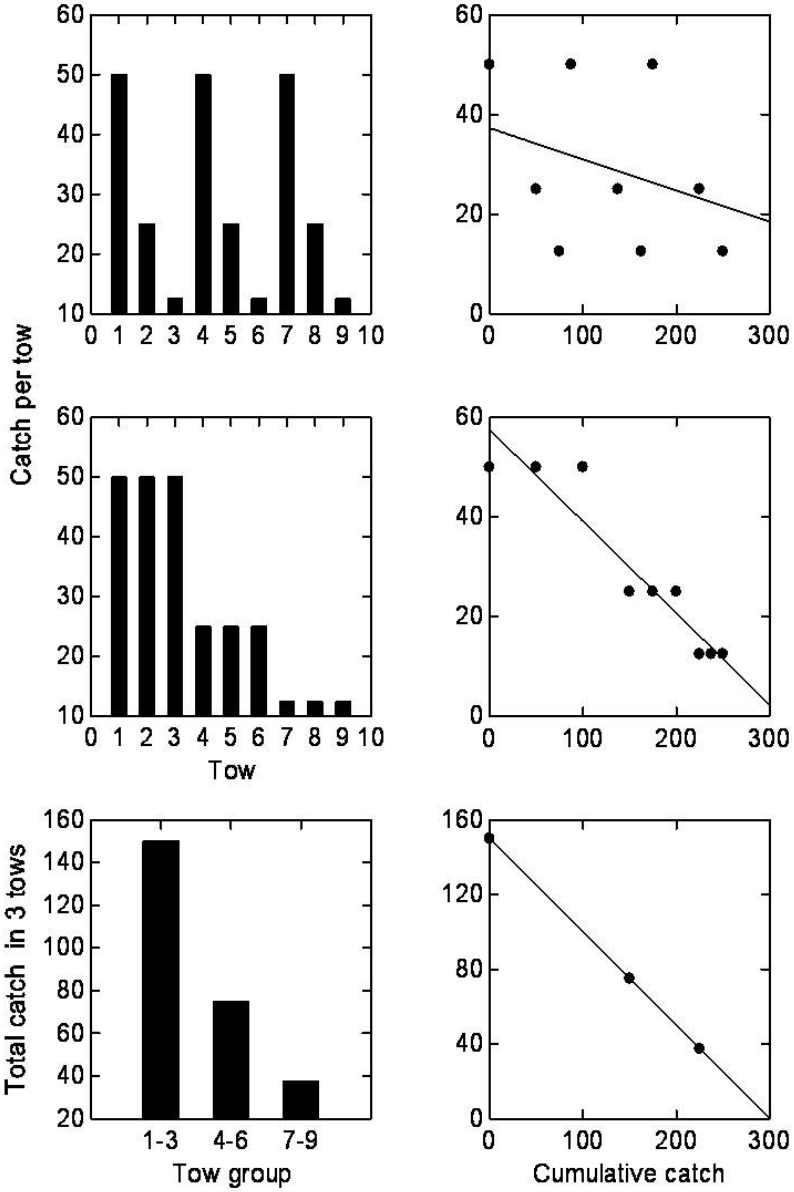




Method 1



Method 2



Toy Example: Why spatial pattern of depletion tows matters.

True Population = 300
True Efficiency=0.5

MLE(1) N=595, Efficiency=0.06

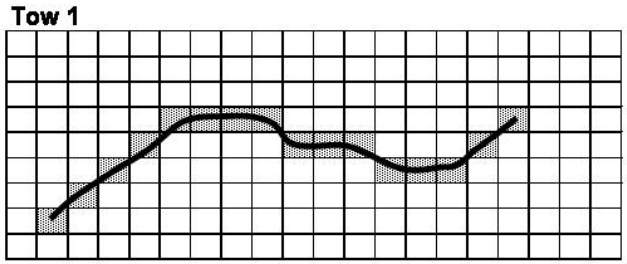
If the catch patterns are incorrectly assumed to be the result of random variability, then MLE estimates are biased.

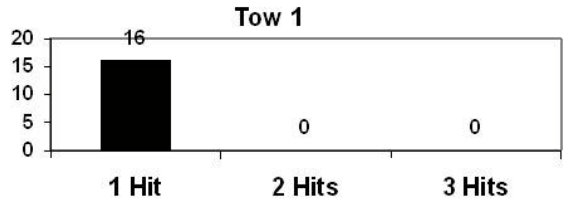
MLE(2) N=312 Efficiency=0.18

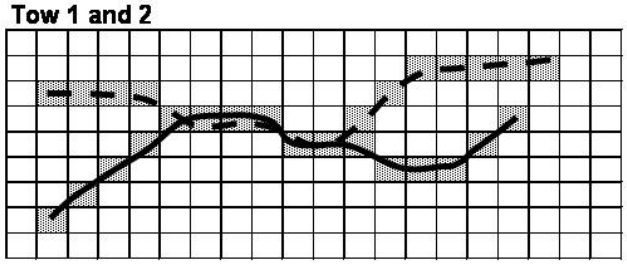
Pooling of samples can work *IF* you know how to aggregate the tows.

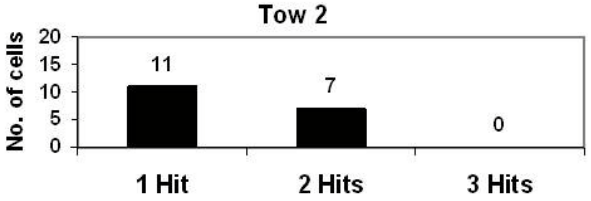
MLE(3) N=300 Efficiency=0.50

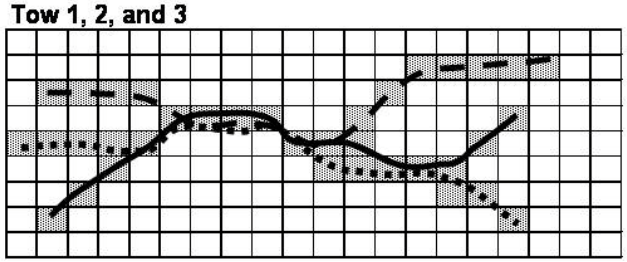
Consider a spatial pattern where the precision of sampling cannot be controlled but it is possible to "know" where the gear is, after the sample is taken. Clearly, the expected catch is a function of the amount of overlap between successive tows.

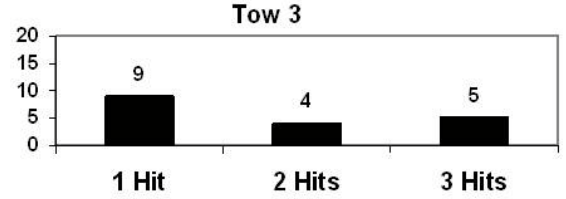












To solve the problem, it is necessary to recast the depletion problem in terms of density per unit area, AND to consider a sample as a string of quadrats or patches.

N_0	$D_0 = N_0/A$
$N_1 = N_0 - C_1$ $= N_0 - e\,(a/A)N_0$ $= N_0\,(1 - e\,(a/A))$	$D_1 = N_1/A$ $= (N_0/A)\,(1-e\,(a/A))$
$N_2 = N_1 - C_2$ $= N_0(1-e(a/A)) - e\,(a/A)N_0\,(1-e(a/A))$ $= N_0\,(1-e(a/A))\,(1-e(a/A))$ $= N_0\,(1-e(a/A))^2$	$D_2 = N_2/A$ $= (N_0/A)\,(1-e\,(a/A))^2$ $= D_0\,(1-e\,(a/A))^2$
$N_3 = N_2 - C_3$ $= N_0\,(1-e(a/A))^2 - e\,(a/A)N_0\,(1-e\,(a/A))^2$ $= N_0\,(1-e(a/A))^2\,(1-e(a/A))$ $= N_0\,(1-e(a/A))^3$	$D_3 = N_3/A$ $= (N_0/A)\,(1-e\,(a/A))^3$ $= D_0\,(1-e\,(a/A))^3$

The expected catch as a function of patch density.

- Via the miracle of recursive pluggation

$$(3) \qquad E(C_j) = e\, a\, D_o \left(1 - e\left(\frac{a}{A}\right)\right)^{j-1}$$

- Now we need to extend this concept to a tow where a tow consists of a set of contiguous patches linked together. Record the number of times that a tow "hits" a particular patch. The first tow will consist of a set of quadrats with one hit each. The second tow, if it exactly overlaps the first, will consist of a set of quadrats with two hits each. Otherwise it will have a mix of 1 and 2 hit cells. The third tow can have some mix of 1, 2 or 3 hits. Etc. With some algebra you get

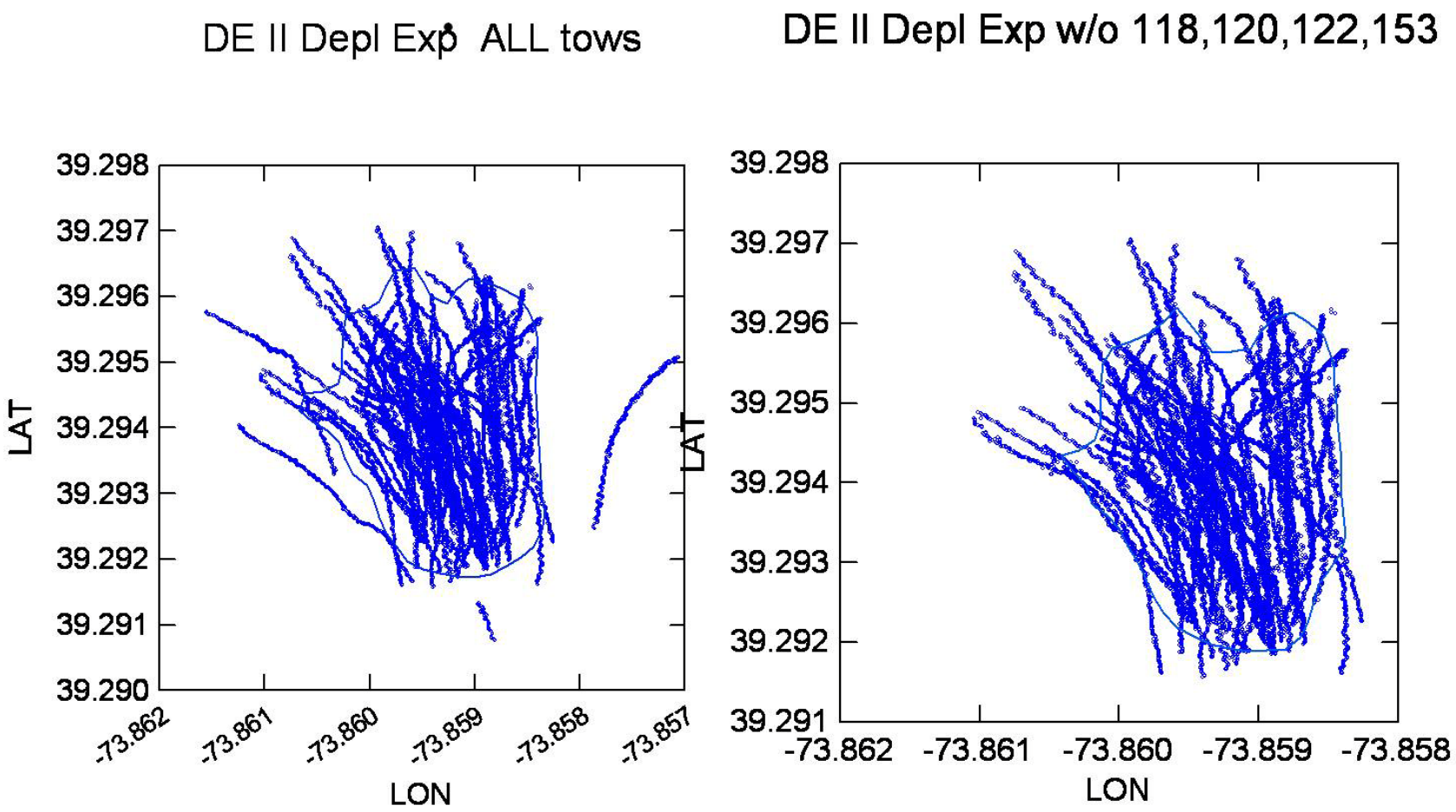
$$(4) \qquad E(C_i) = e\, a_i\, D_o \sum_{j=1}^{i} f_{i,j} (1 - e\gamma)^{j-1}$$

OK, where did the gamma come from and what is $f_{i,j}$?

$$(4) \qquad E(C_i) = e\, a_i\, D_o \sum_{j=1}^{i} f_{i,j} (1 - e\gamma)^{j-1}$$

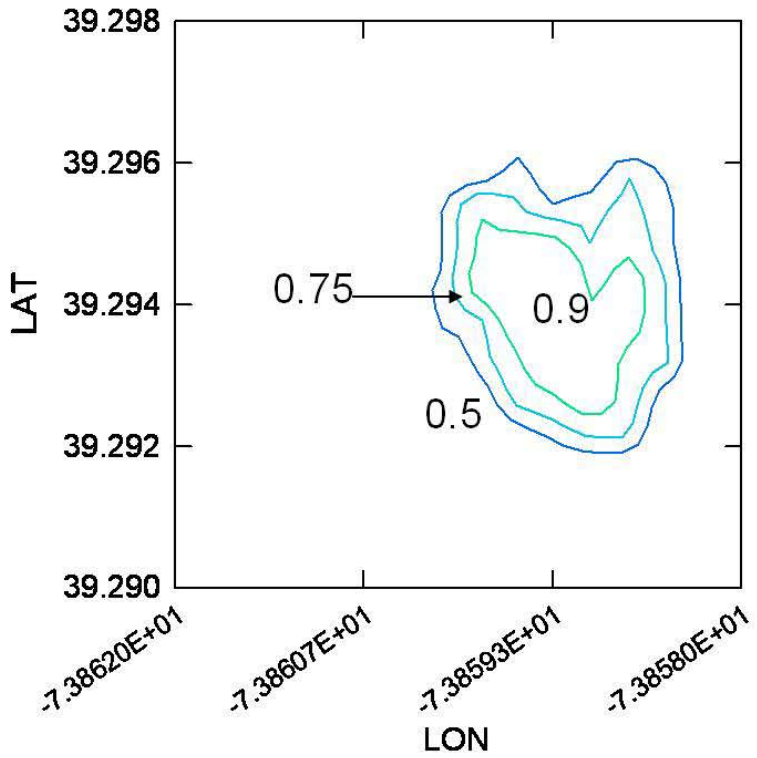
- $f_{i,j}$ is the fraction of a tow of swept area a_i that is hit j times
- Gamma is ratio of the sampling device width over the width of the patch. If the width of sampler is ½ the width of the cell, then gamma =0.5. Thus the patch size can increase in response to increasing uncertainty in the actual position of the sampling gear.
- Gamma can also be used to account for indirect effects of sampling which may occur in a variety of ways. More on this later.

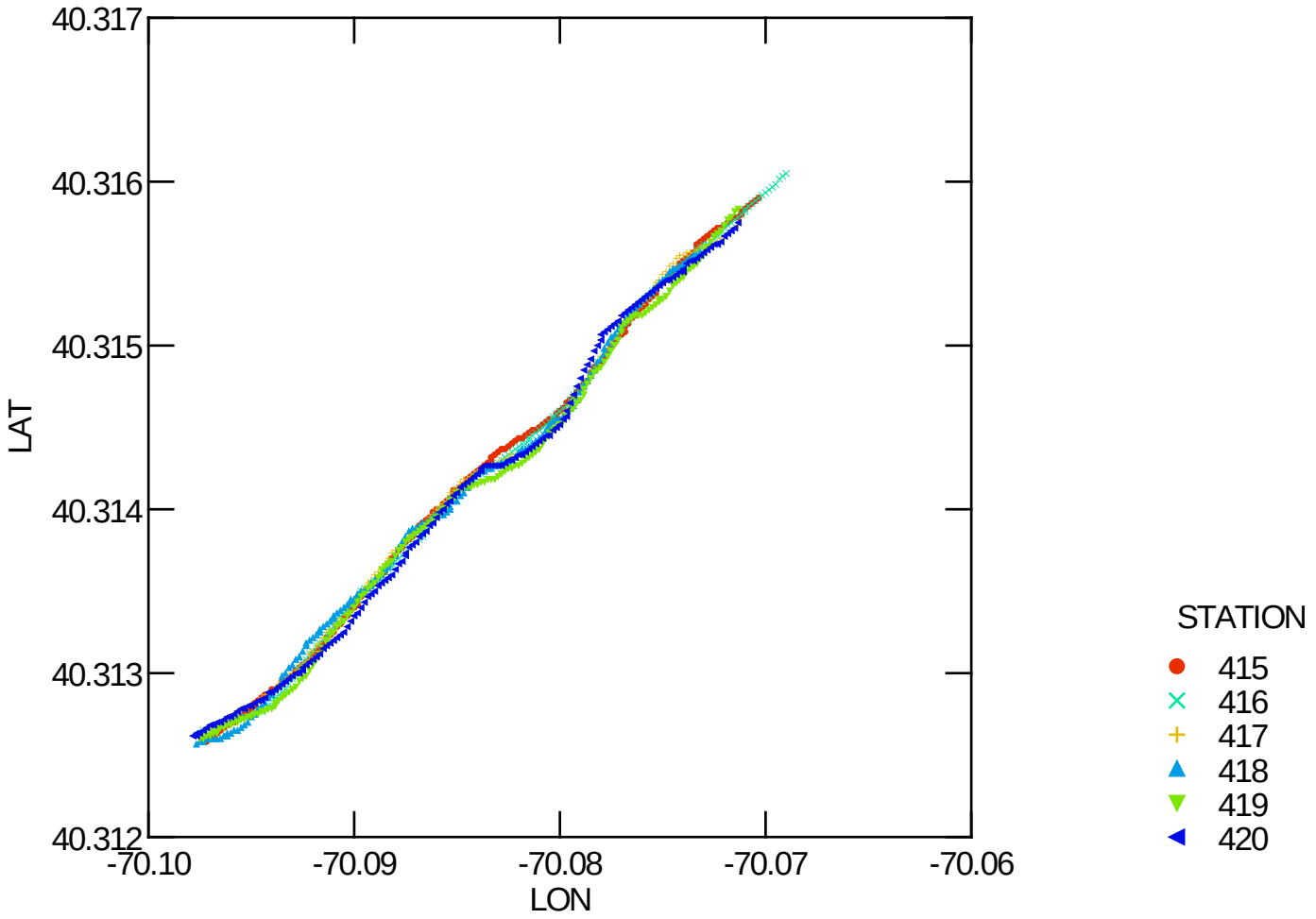
Deriving the expected catch for a set of randomly overlapping tows. A depletion experiment on DE II

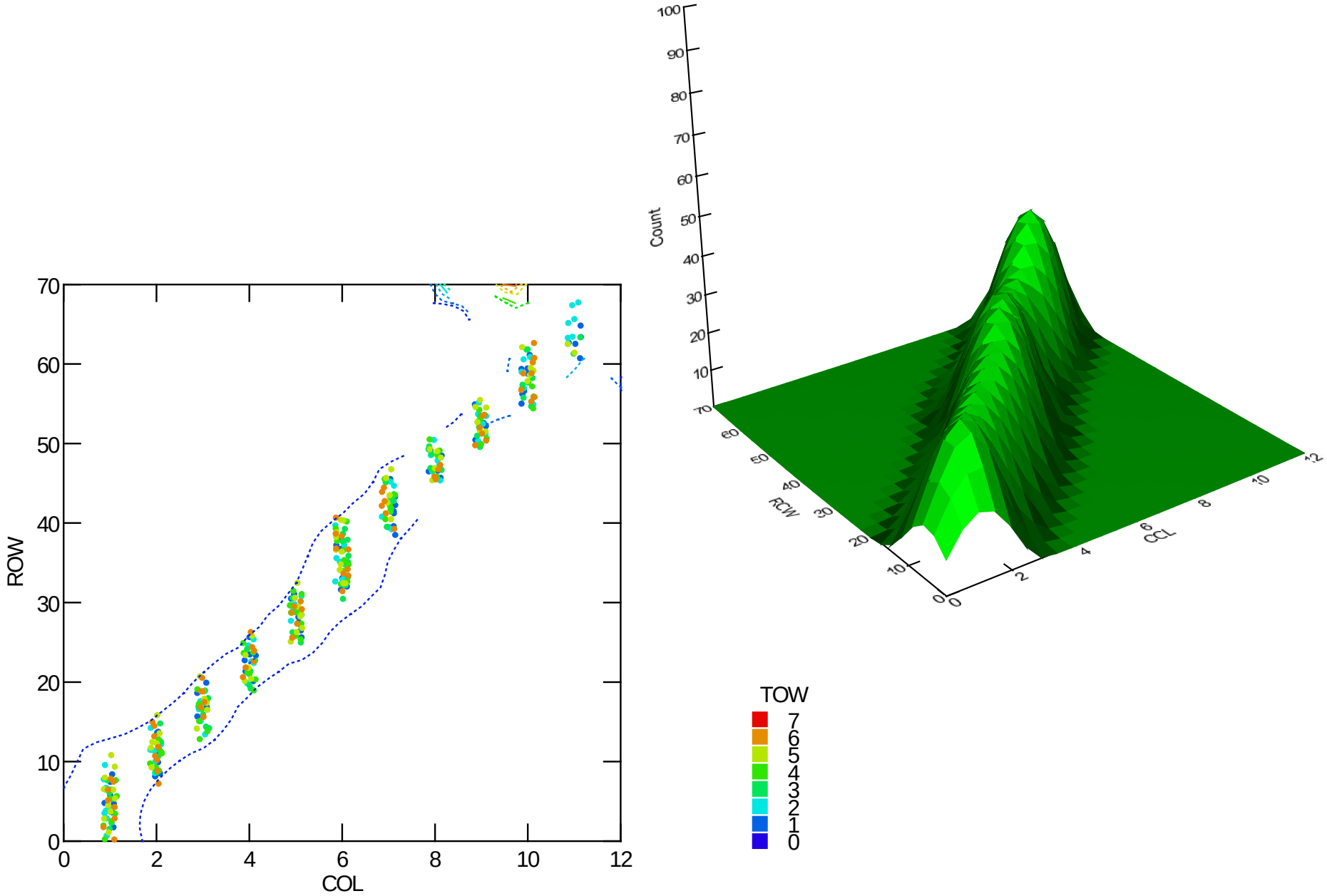


A kernel density can be used to illustrate to concentration of sampling intensity.

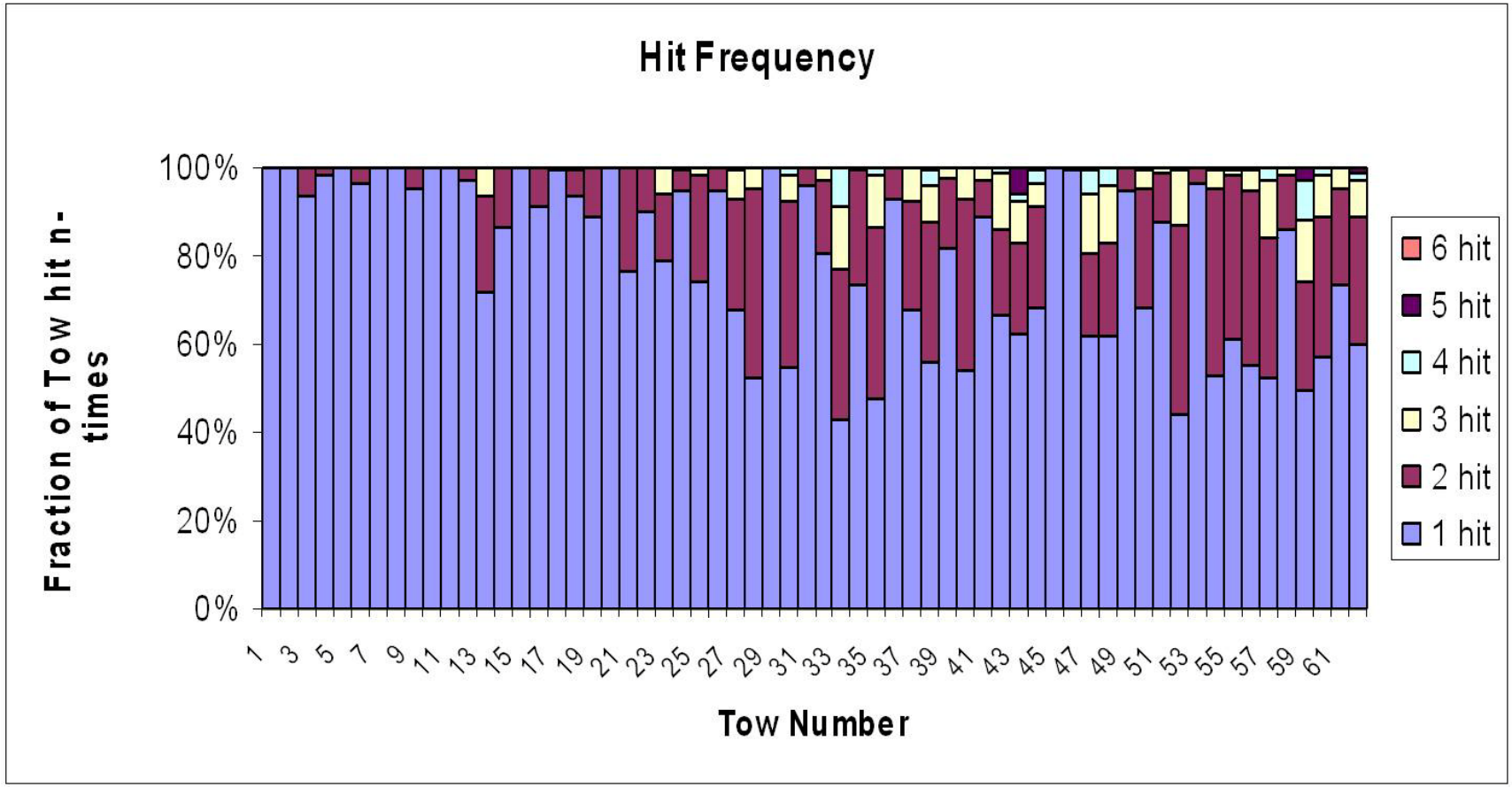
- DE II Depl Exp w/o 118,120,122,153

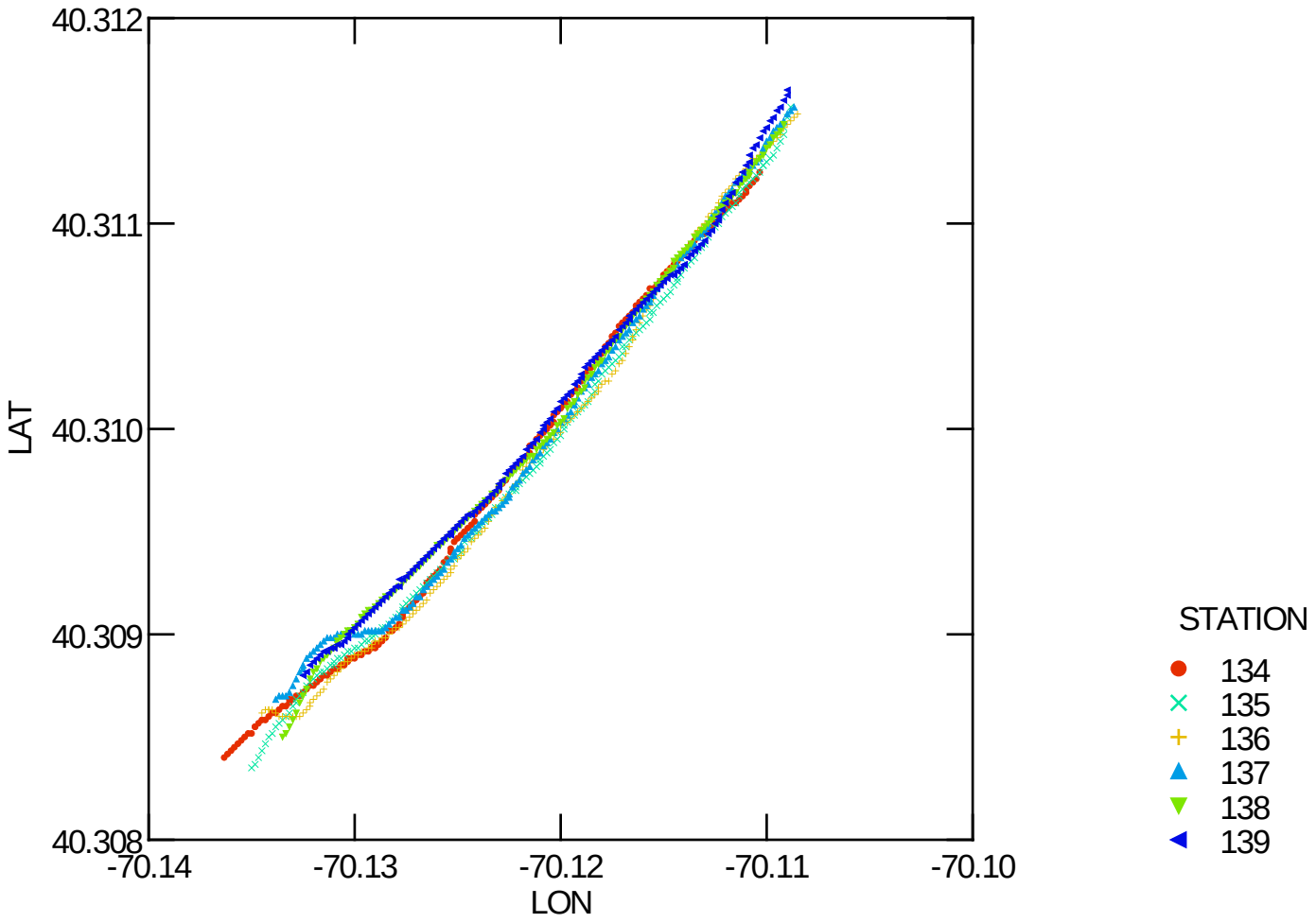


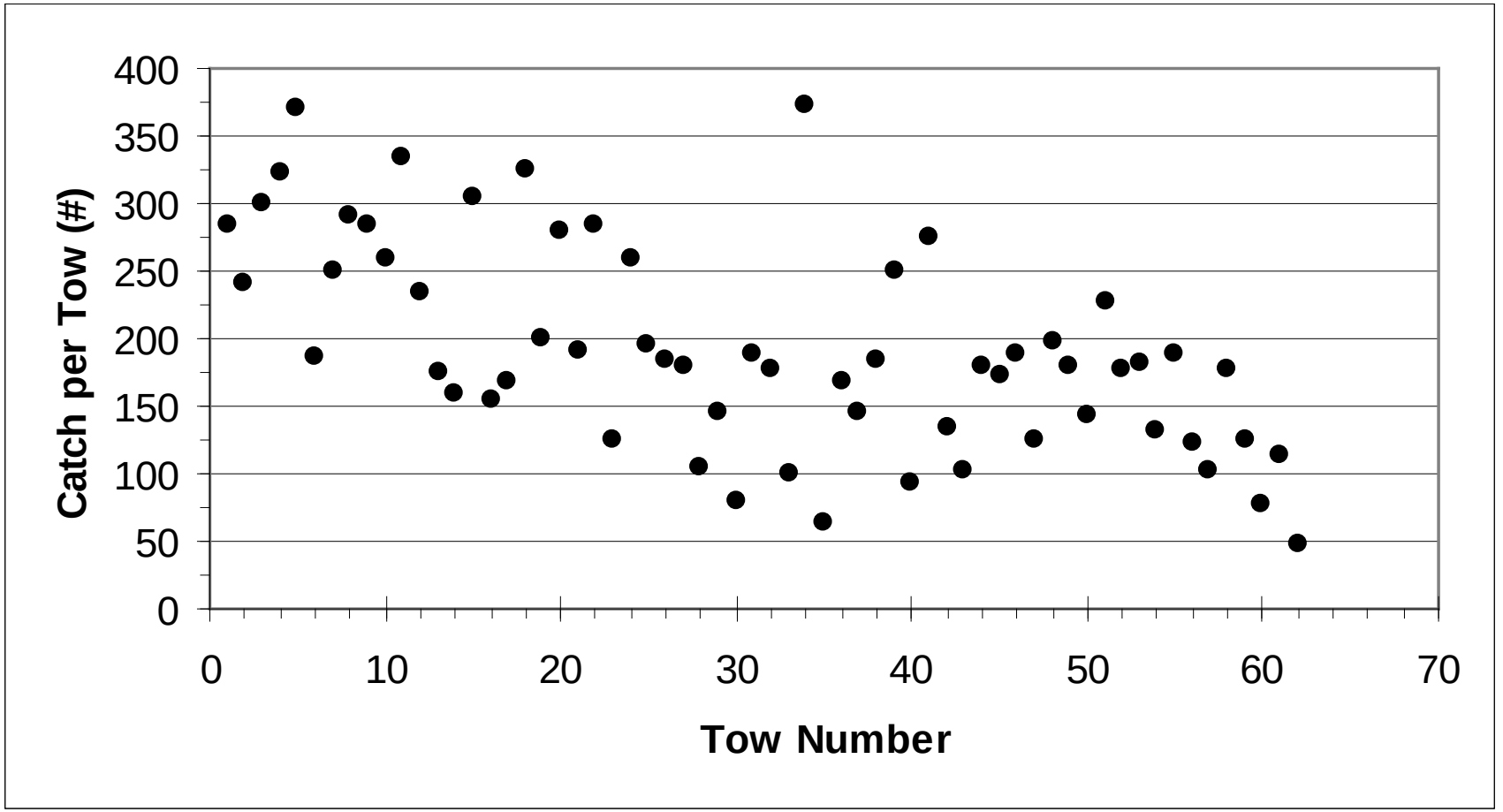

experiment #5
40.317
40.316
40.315
40.314
40.313
40.312
LAT
-70.10
-70.09
-70.08
-70.07
-70.06
LON
STATION
415
416
417
418
419
420

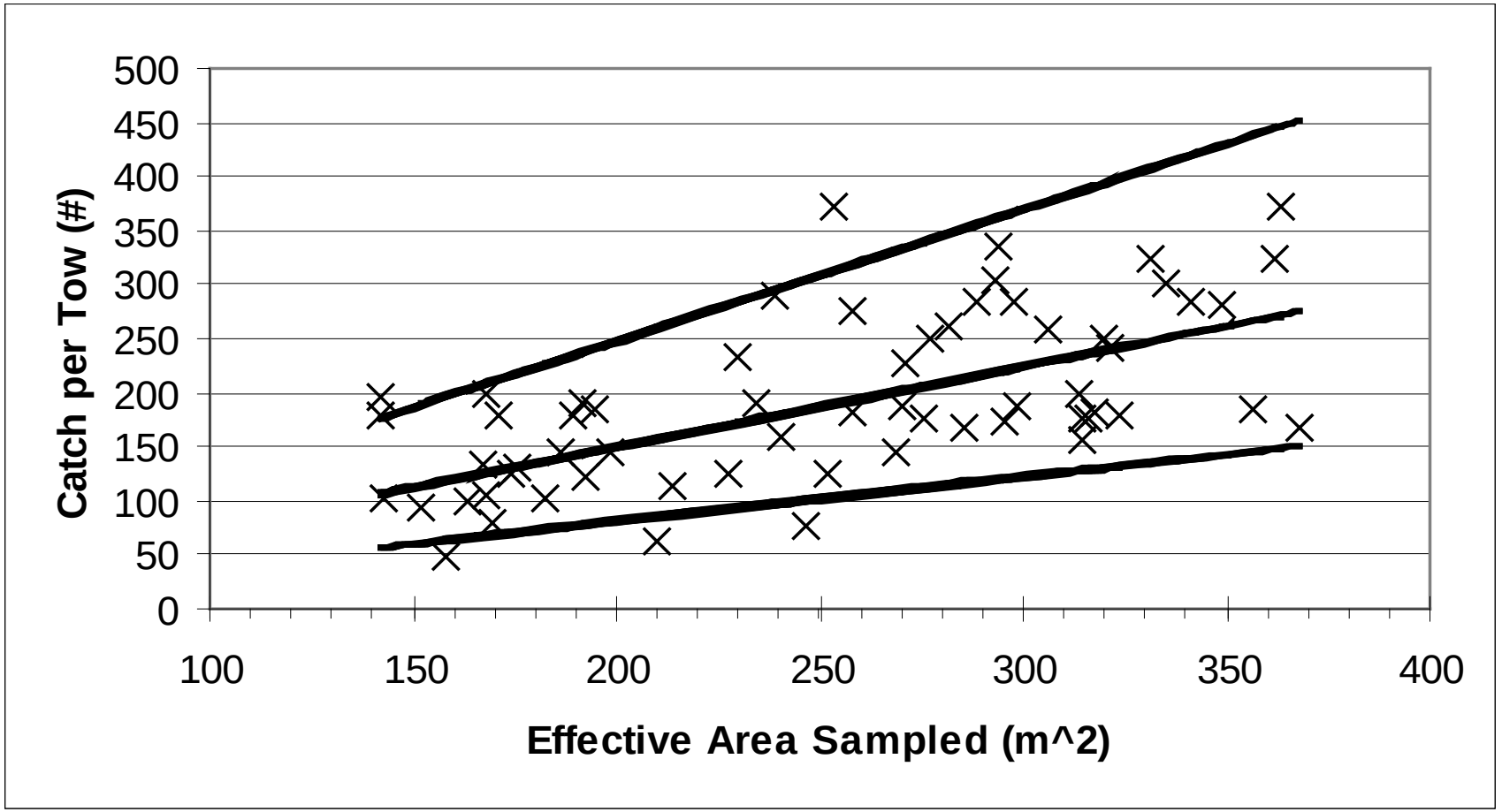

ROW
COL
0
10
20
30
40
50
60
70
0
2
4
6
8
10
12
Count
100
90
80
70
60
50
40
30
20
10
0
ROW
COL
TOW
7
6
5
4
3
2
1
0

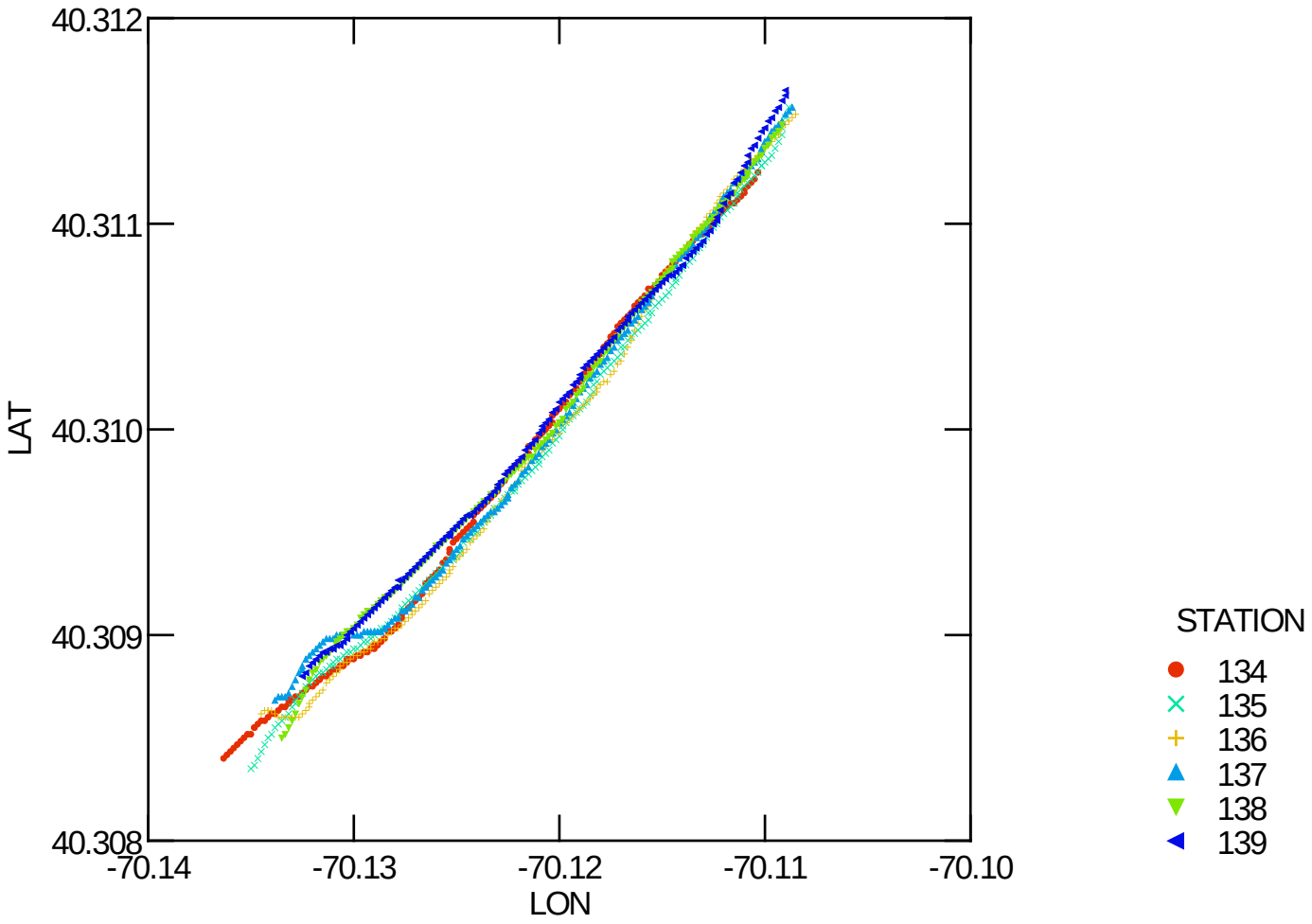
The hit frequency matrix for the Delaware II depletion experiment

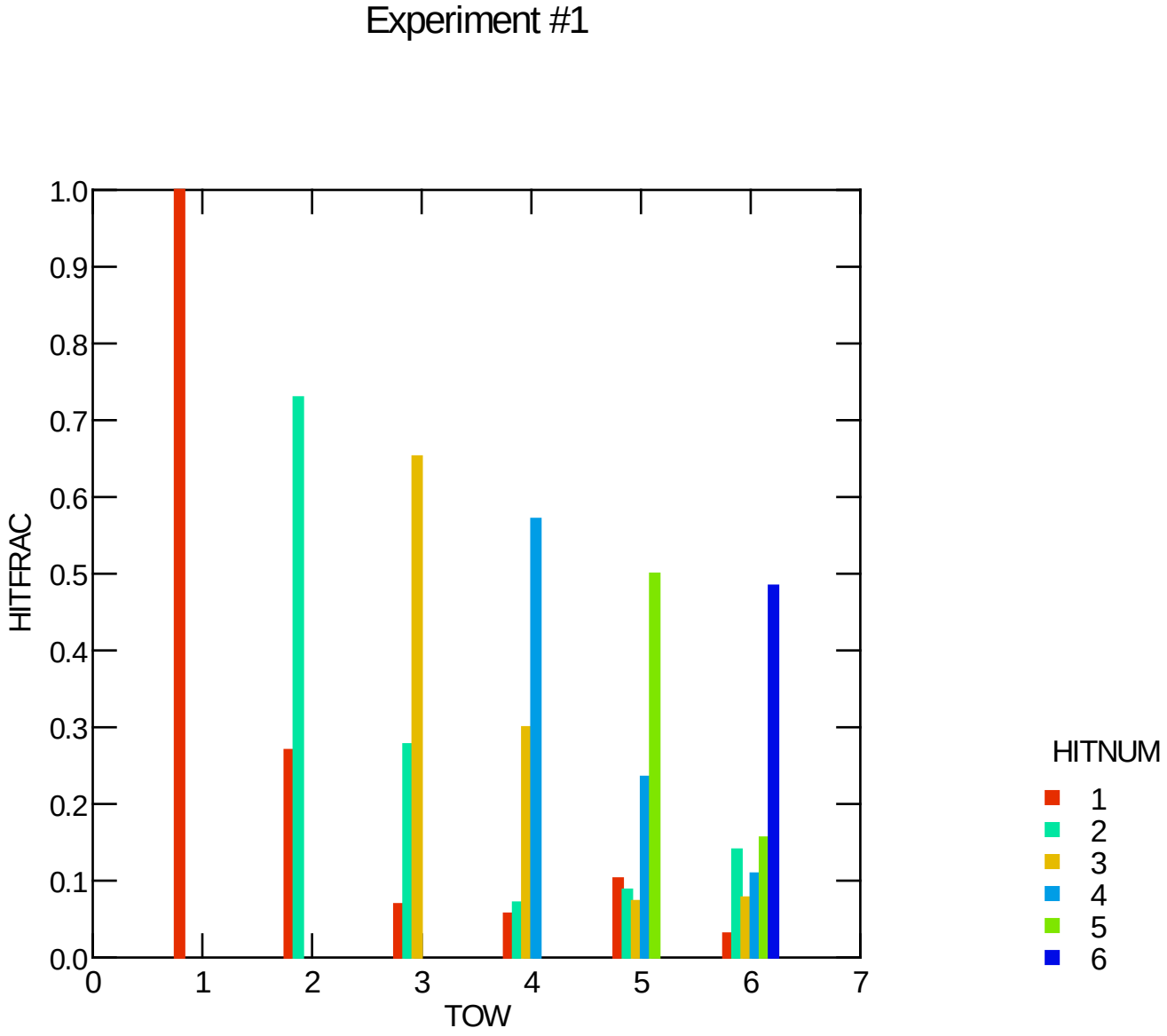



experiment #1
40.312
40.311
40.310
40.309
40.308
LAT
-70.14
-70.13
-70.12
-70.11
-70.10
LON
STATION
134
135
136
137
138
139

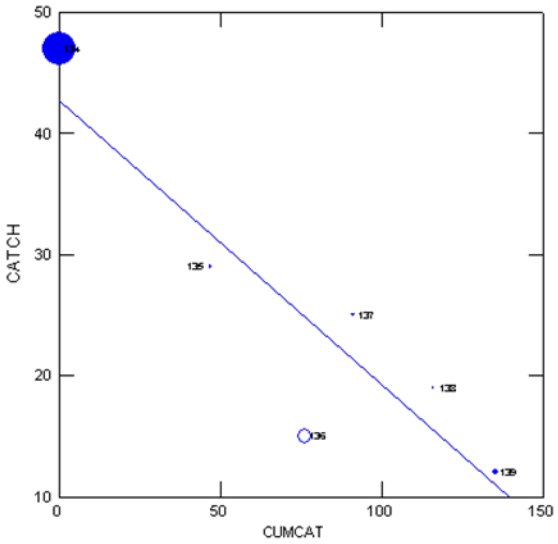

Catch per Tow (#)
400
350
300
250
200
150
100
50
0
0
10
20
30
40
50
60
70
Tow Number

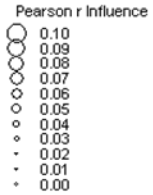

Catch per Tow (#)
500
450
400
350
300
250
200
150
100
50
0
100
150
200
250
300
350
400
Effective Area Sampled (m^2)


experiment #1
40.312
40.311
40.310
40.309
40.308
LAT
-70.14
-70.13
-70.12
-70.11
-70.10
LON
STATION
134
135
136
137
138
139

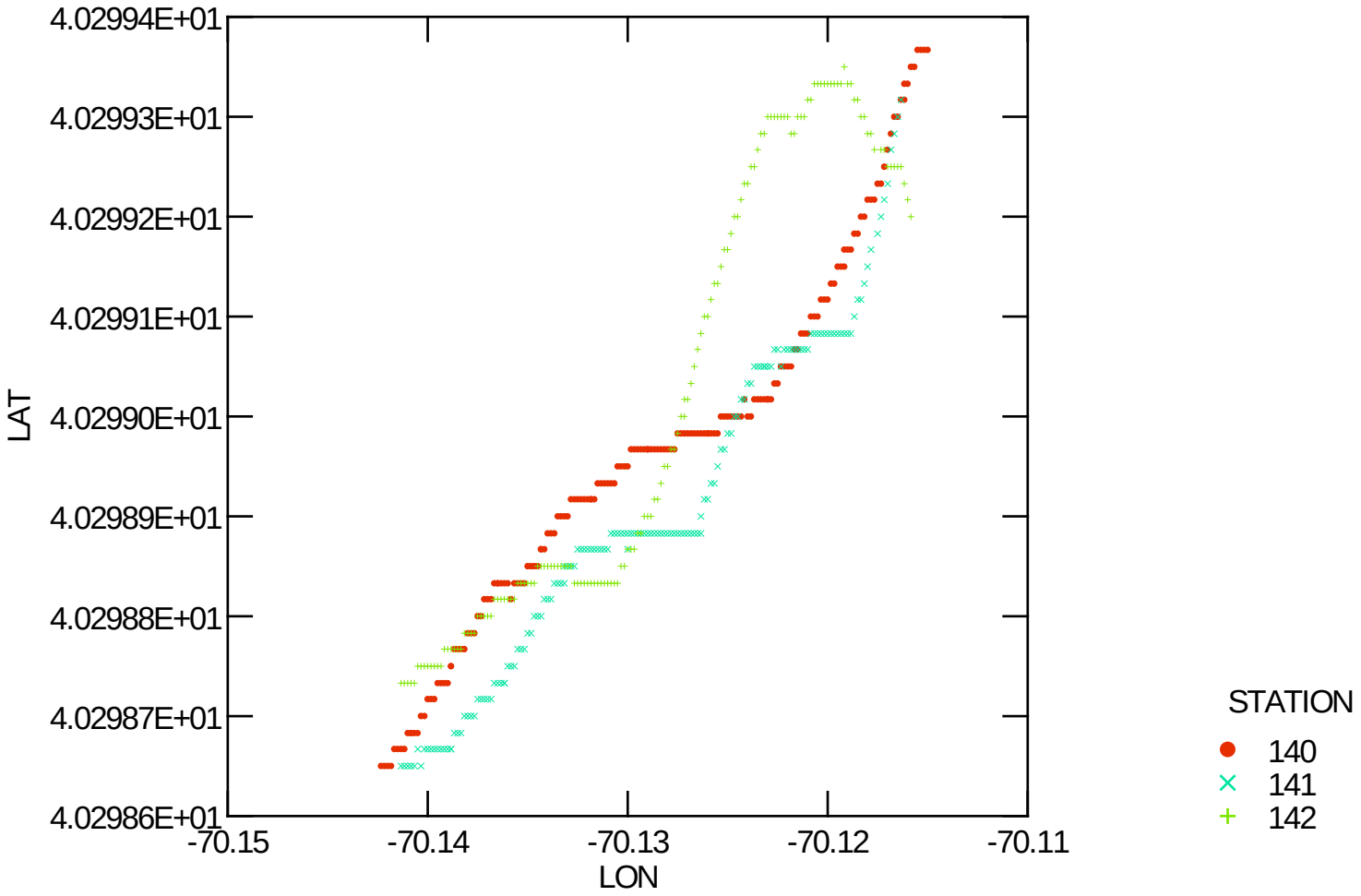

Experiment #1
HITFRAC
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
TOW
HITNUM
1
2
3
4
5
6

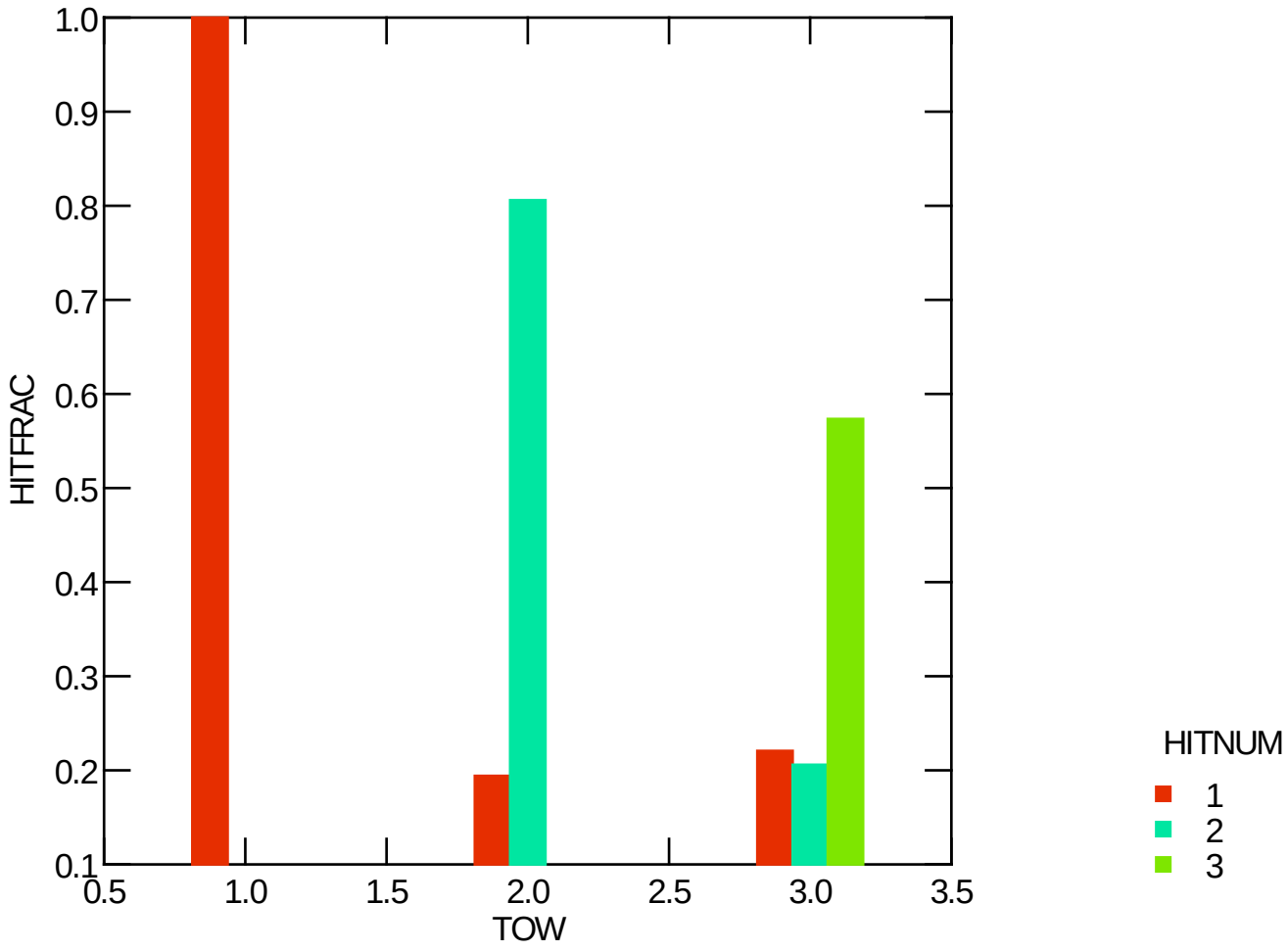
Experiment #1

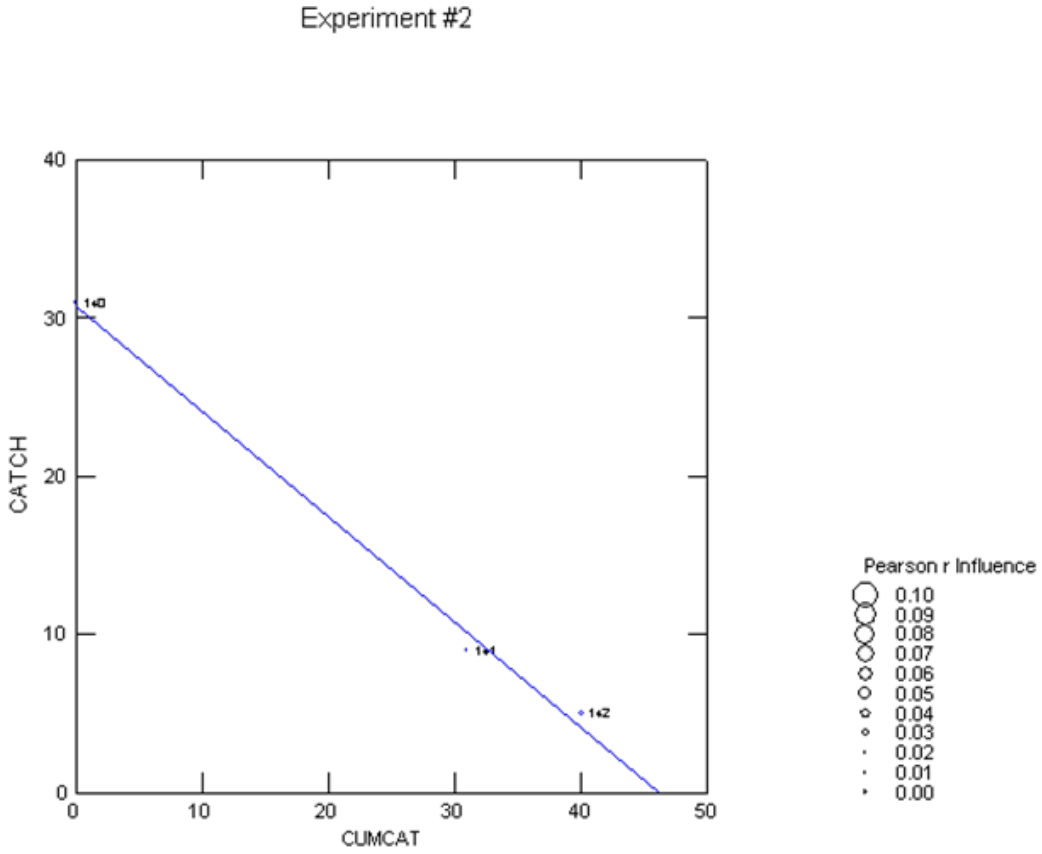




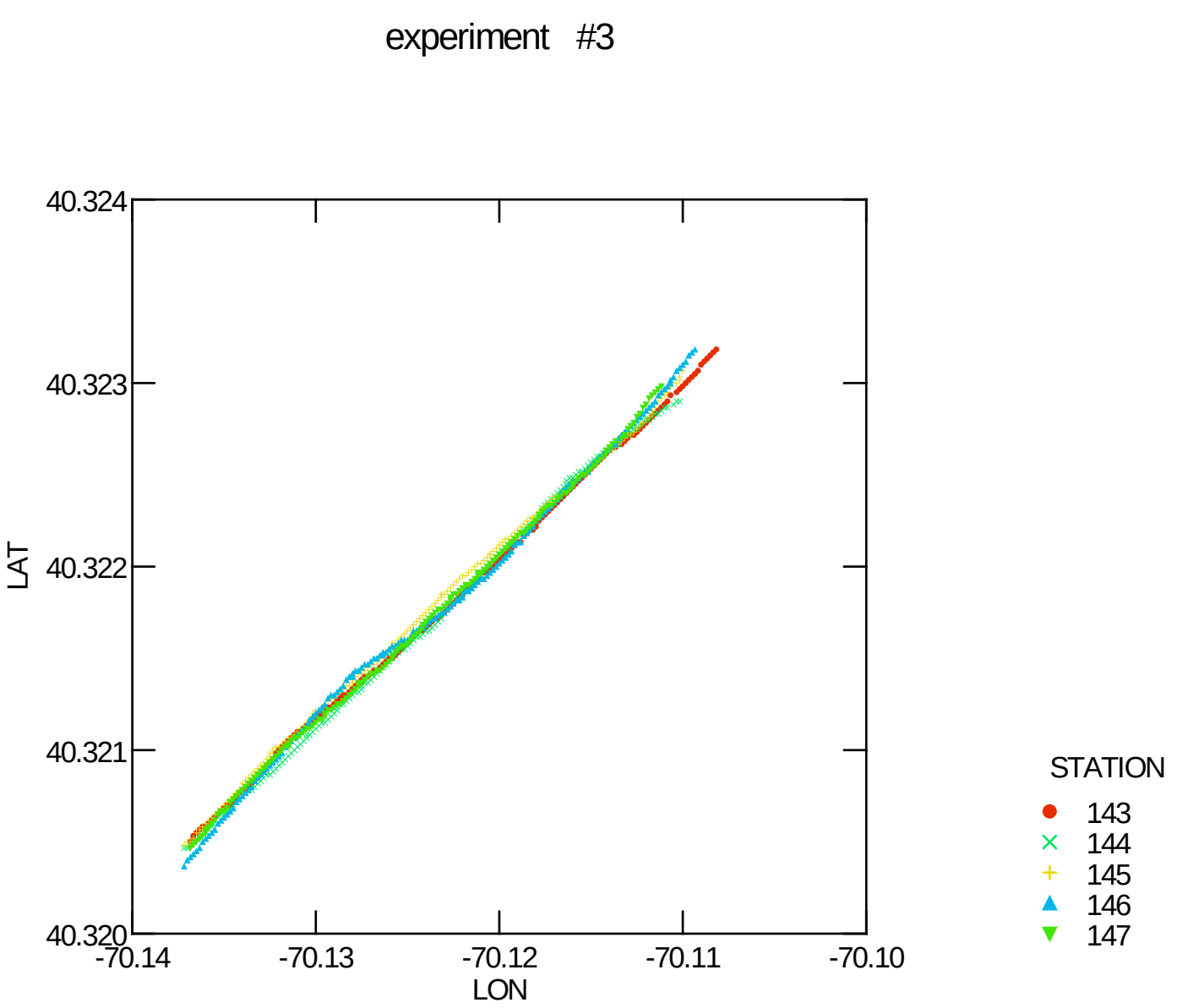
Tow	Area Swept	Efftv Area	C_5%	C_50%	C_pred	OBS	C_95%
CI 1	566598.0	194434.7	29.	41.	41.	47.	56.
CI 2	572557.0	157118.4	23.	34.	33.	29.	46.
CI 3	569490.0	120094.9	17.	26.	25.	15.	36.
CI 4	545629.0	90817.8	12.	20.	19.	25.	29.
CI 5	525474.0	78258.5	10.	17.	16.	19.	25.
CI 6	509265.0	62348.7	8.	14.	13.	12.	21.

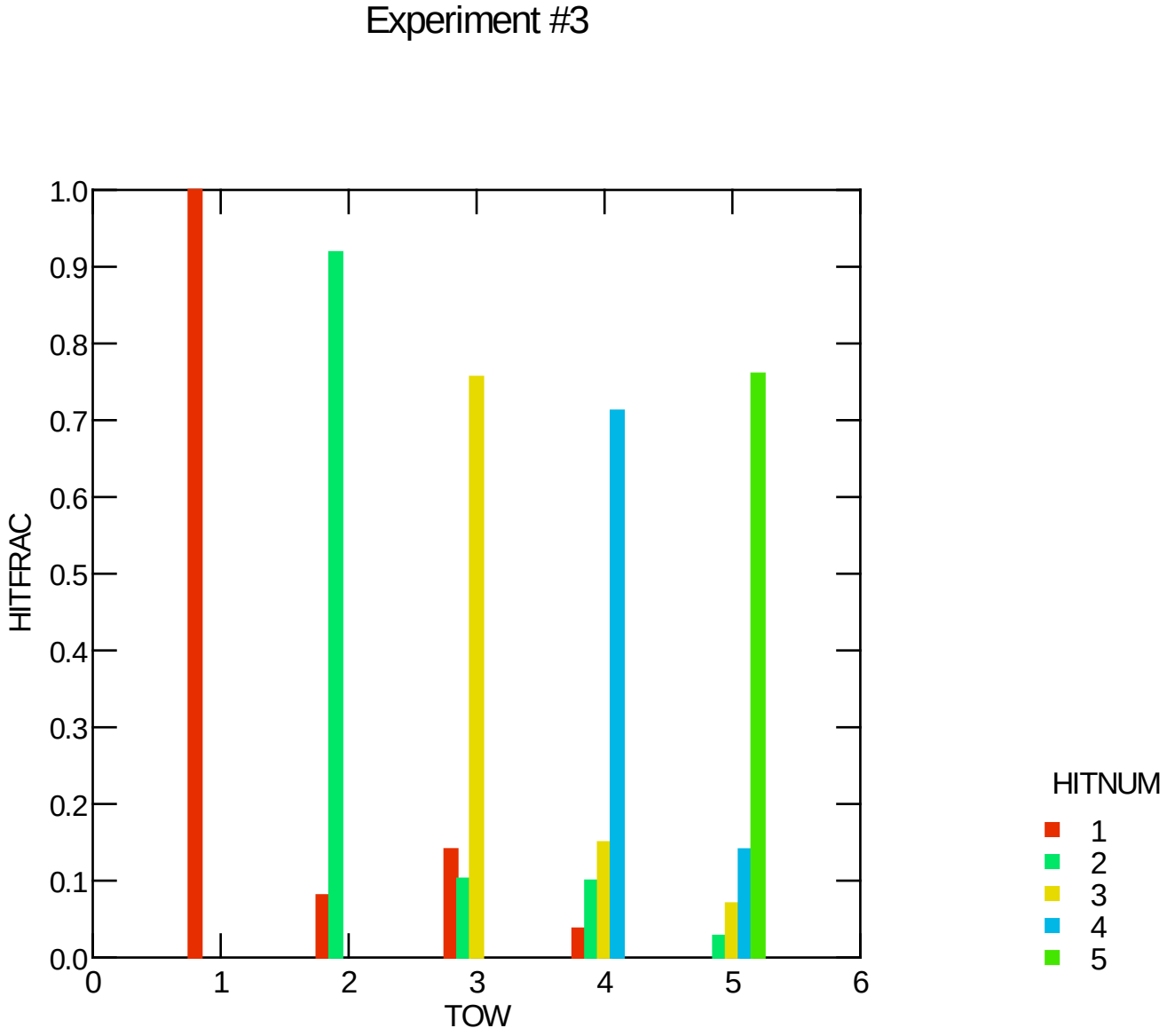

experiment #2
4.02994E+01
4.02993E+01
4.02992E+01
4.02991E+01
4.02990E+01
4.02989E+01
4.02988E+01
4.02987E+01
4.02986E+01
LAT
-70.15
-70.14
-70.13
-70.12
-70.11
LON
STATION
140
141
142

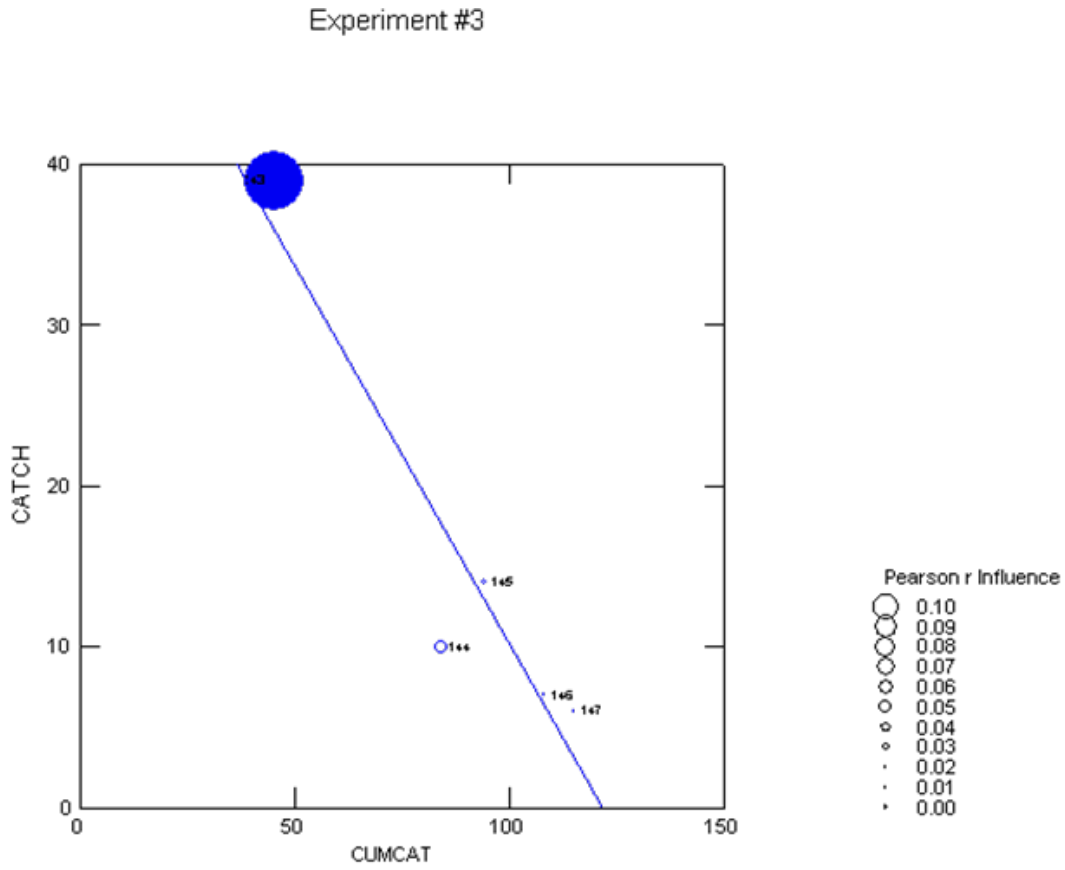

Experiment #2
HITFRAC
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.5
1.0
1.5
2.0
2.5
3.0
3.5
TOW
HITNUM
1
2
3



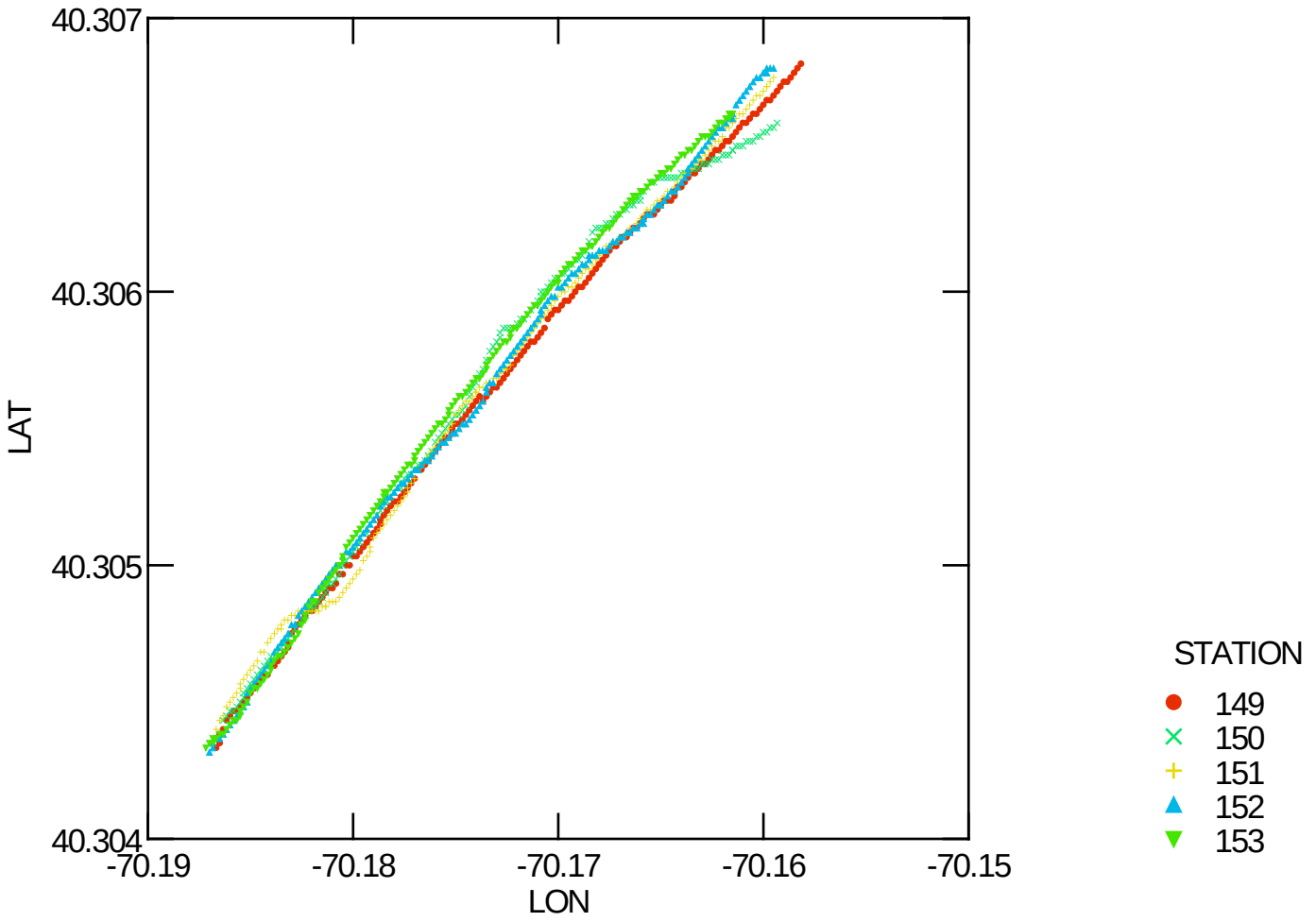
Tow	Area Swept	Efftv Area	C_5%	C_50%	C_pred	OBS	C_95%
CI 1	590158.0	560650.0	20.	28.	28.	31.	38.
CI 2	533897.0	196483.1	6.	10.	10.	9.	16.
CI 3	545551.0	157031.4	4.	9.	8.	5.	14.

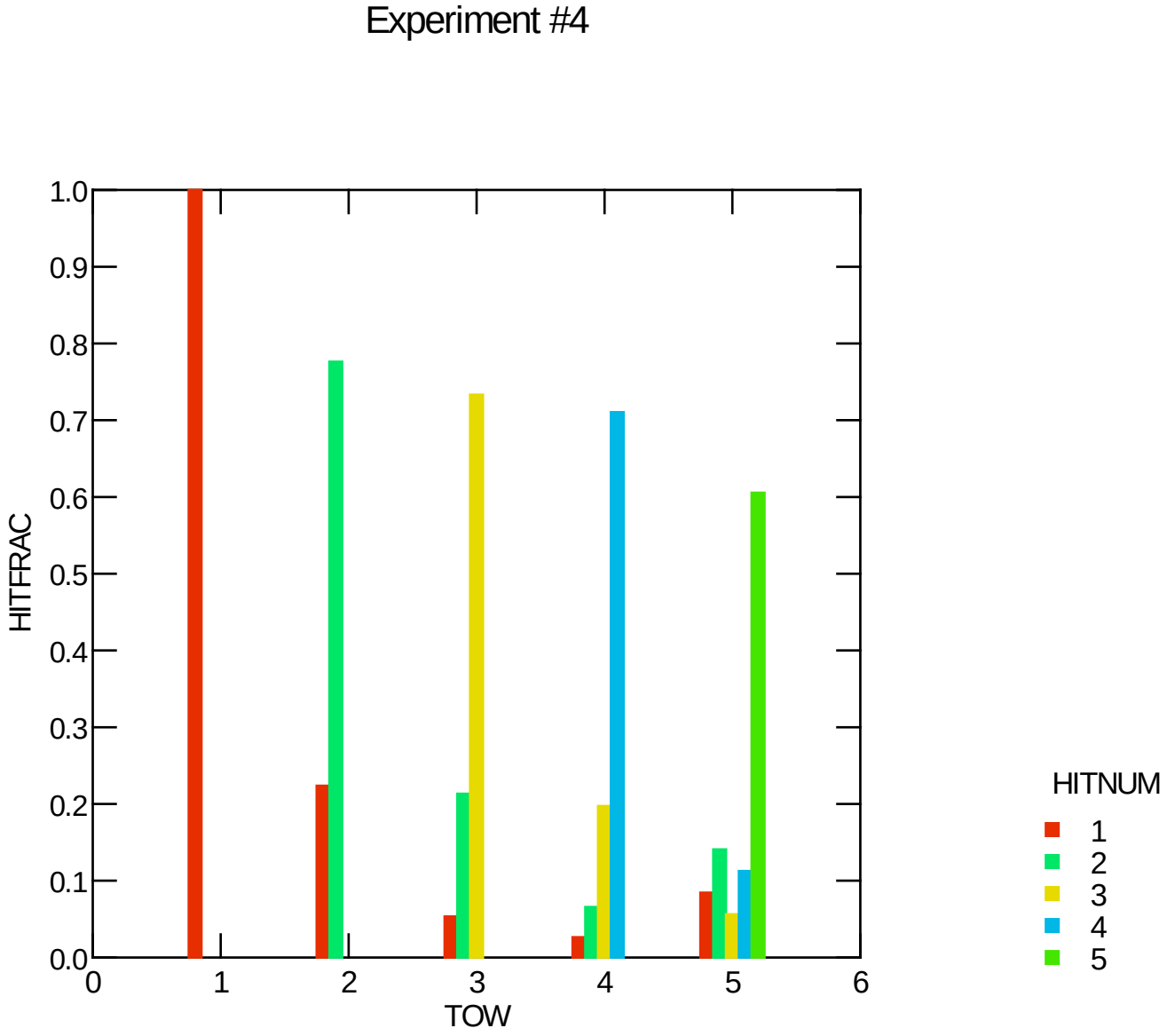

experiment #3
40.324
40.323
40.322
40.321
40.320
LAT
-70.14
-70.13
-70.12
-70.11
-70.10
LON
STATION
143
144
145
146
147


Experiment #3
HITFRAC
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
TOW
HITNUM
1
2
3
4
5

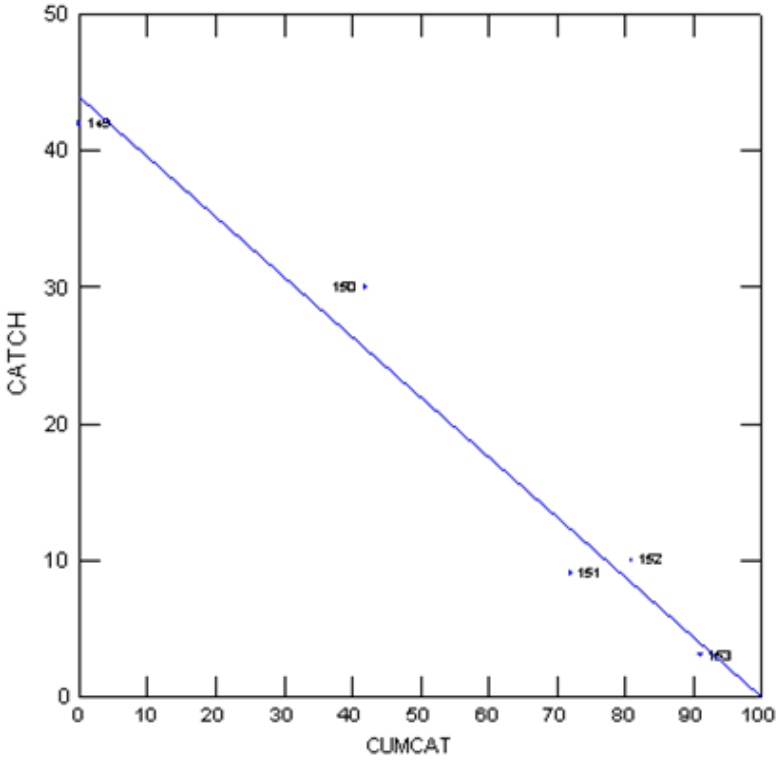


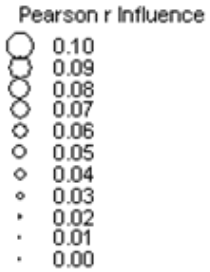
	Tow	Area Swept	Efftv Area	C_5%	C_50%	C_pred	OBS	C_95%
CI	1	634514.0	346094.2	21.	32.	32.	39.	46.
CI	2	590274.0	192848.7	11.	18.	18.	10.	28.
CI	3	597123.0	143012.4	8.	14.	13.	14.	22.
CI	4	612709.0	89918.2	5.	9.	8.	7.	15.
CI	5	562786.0	43035.3	2.	5.	4.	6.	9.


experiment #4
40.307
40.306
40.305
40.304
LAT
-70.19
-70.18
-70.17
-70.16
-70.15
LON
STATION
149
150
151
152
153

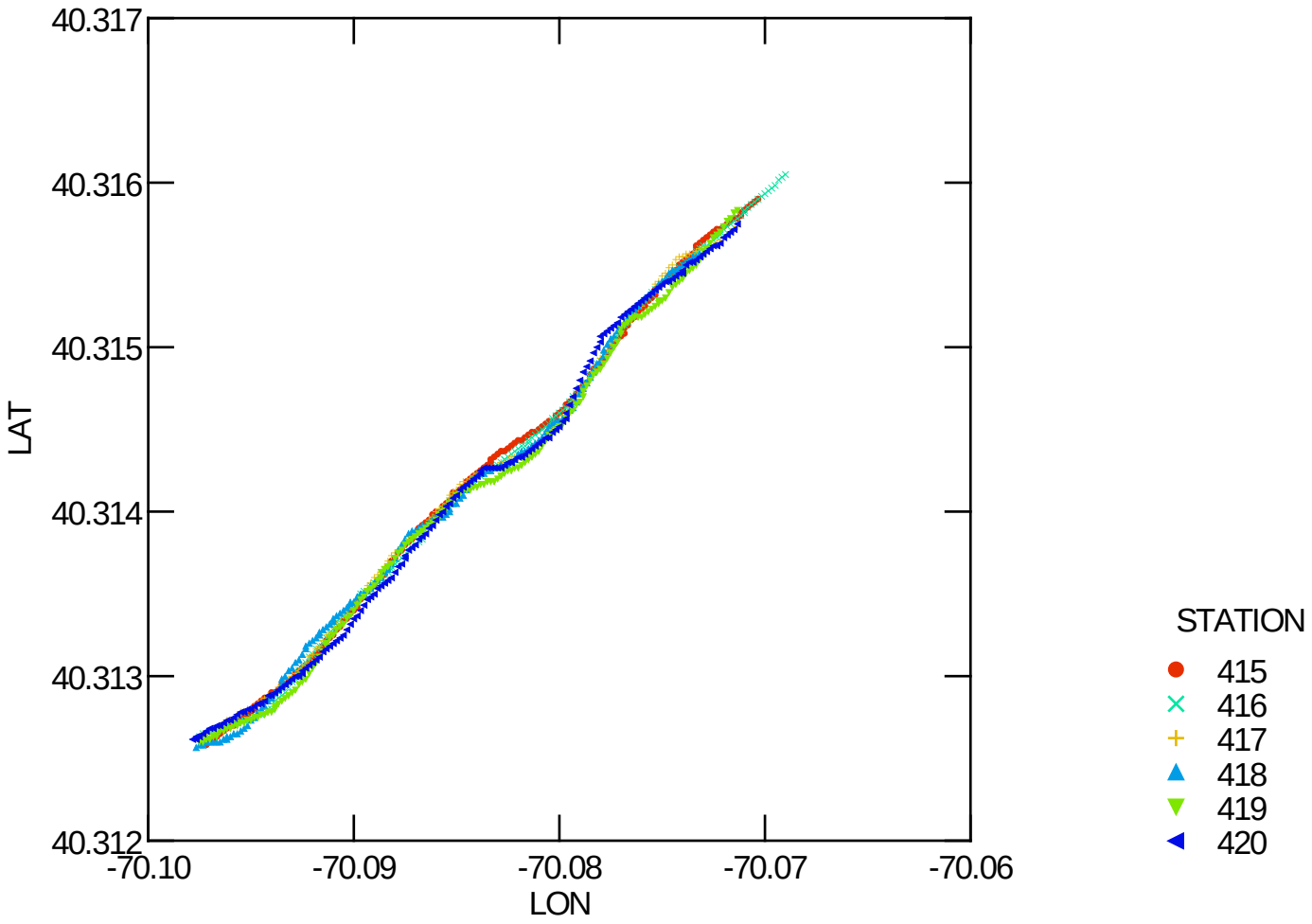

Experiment #4
HITFRAC
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
TOW
HITNUM
1
2
3
4
5

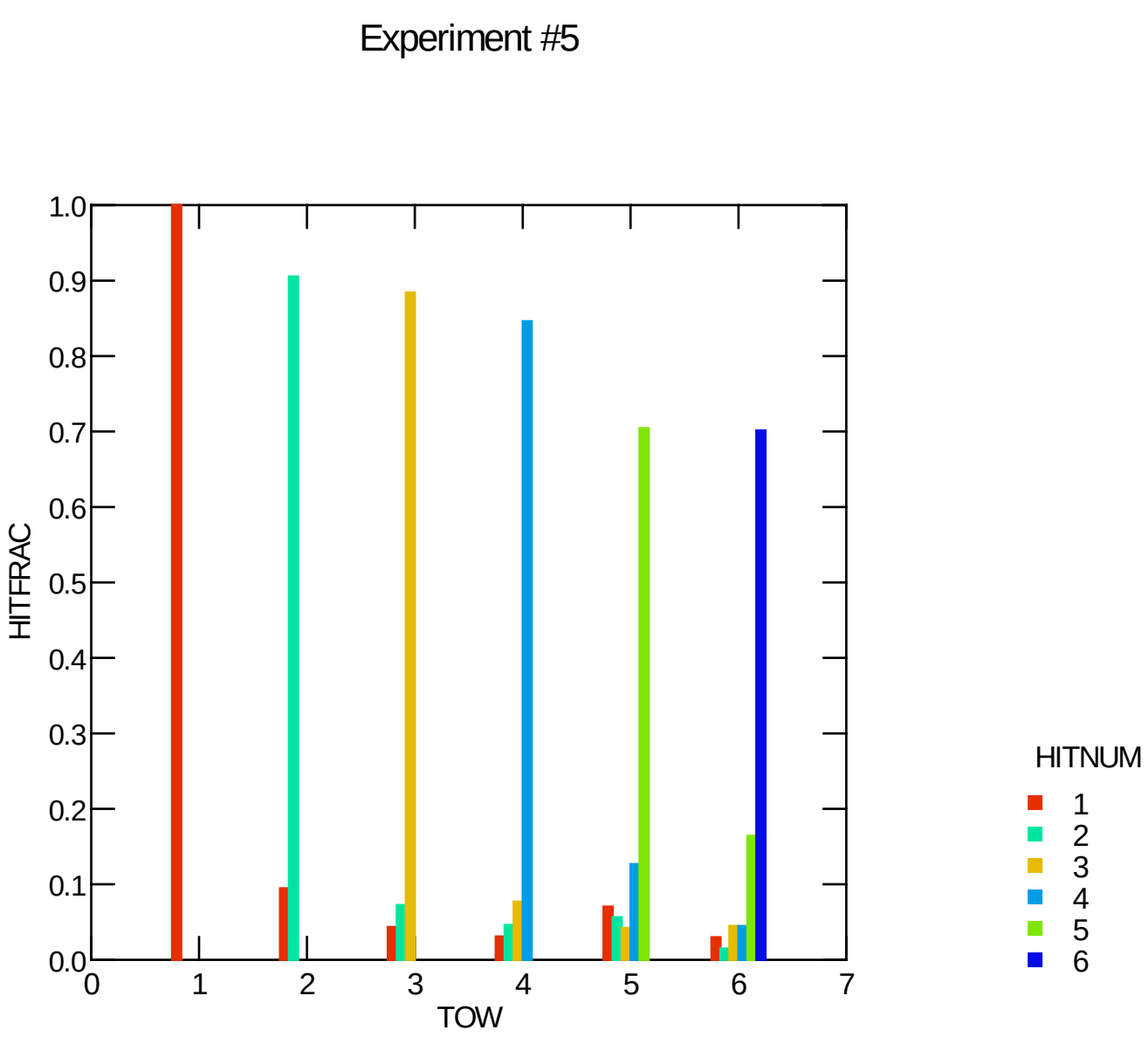
Experiment #4



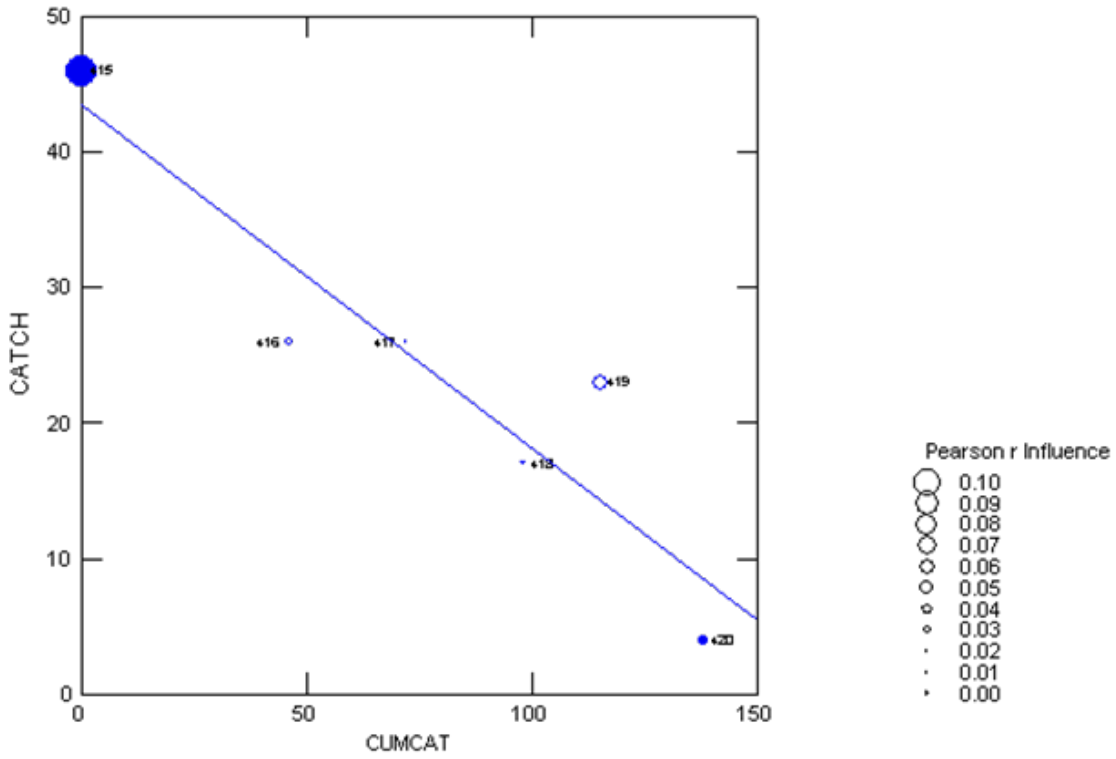


Tow	Area Swept	Efftv Area	C_5%	C_50%	C_pred	OBS	C_95%
CI 1	623017.0	425046.3	34.	44.	43.	42.	56.
CI 2	586683.0	230645.1	17.	24.	24.	30.	33.
CI 3	592784.0	121917.5	8.	13.	12.	9.	20.
CI 4	603661.0	67315.5	4.	8.	7.	10.	12.
CI 5	561273.0	75208.0	4.	9.	8.	3.	14.

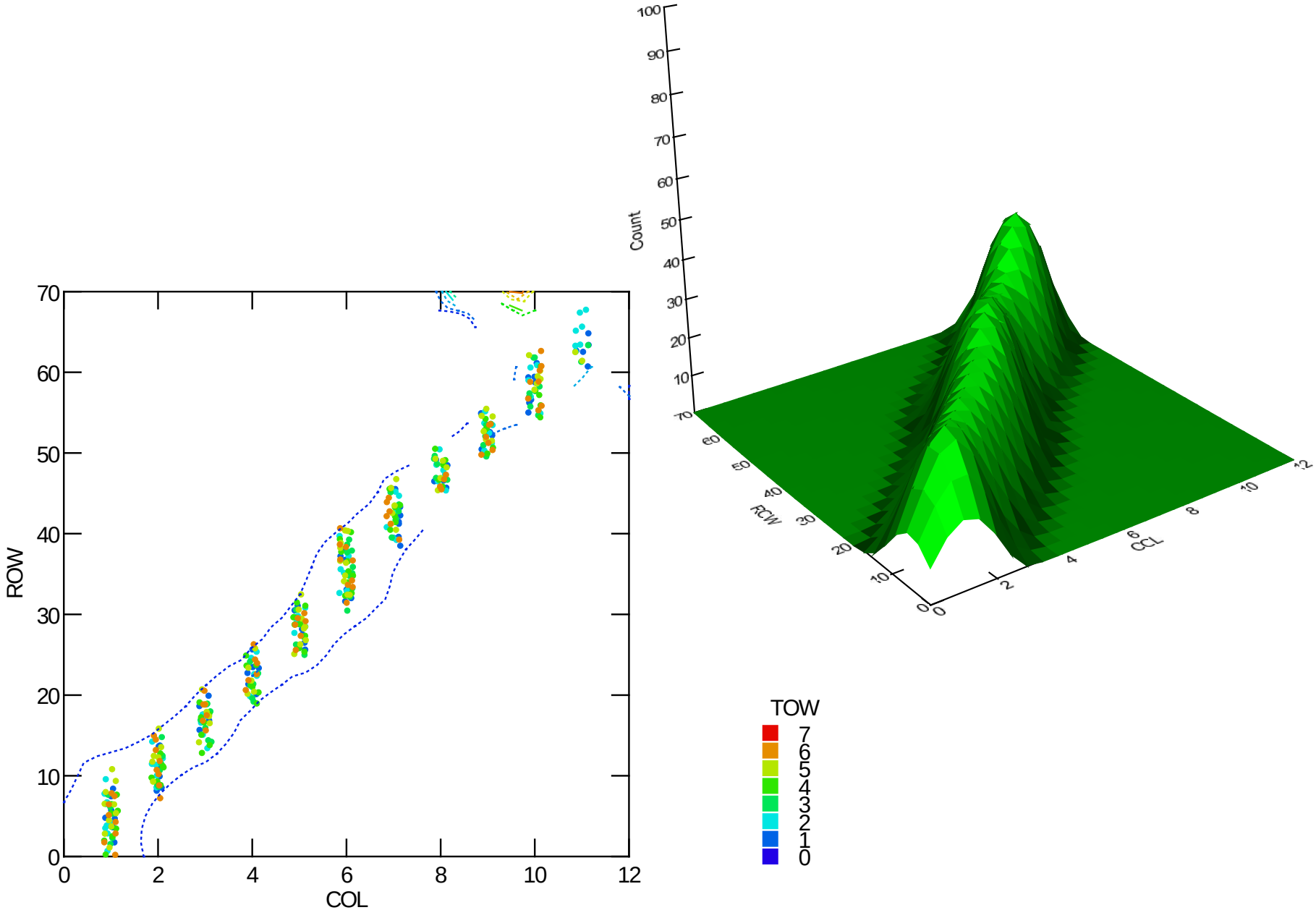

experiment #5
40.317
40.316
40.315
40.314
40.313
40.312
LAT
-70.10
-70.09
-70.08
-70.07
-70.06
LON
STATION
415
416
417
418
419
420

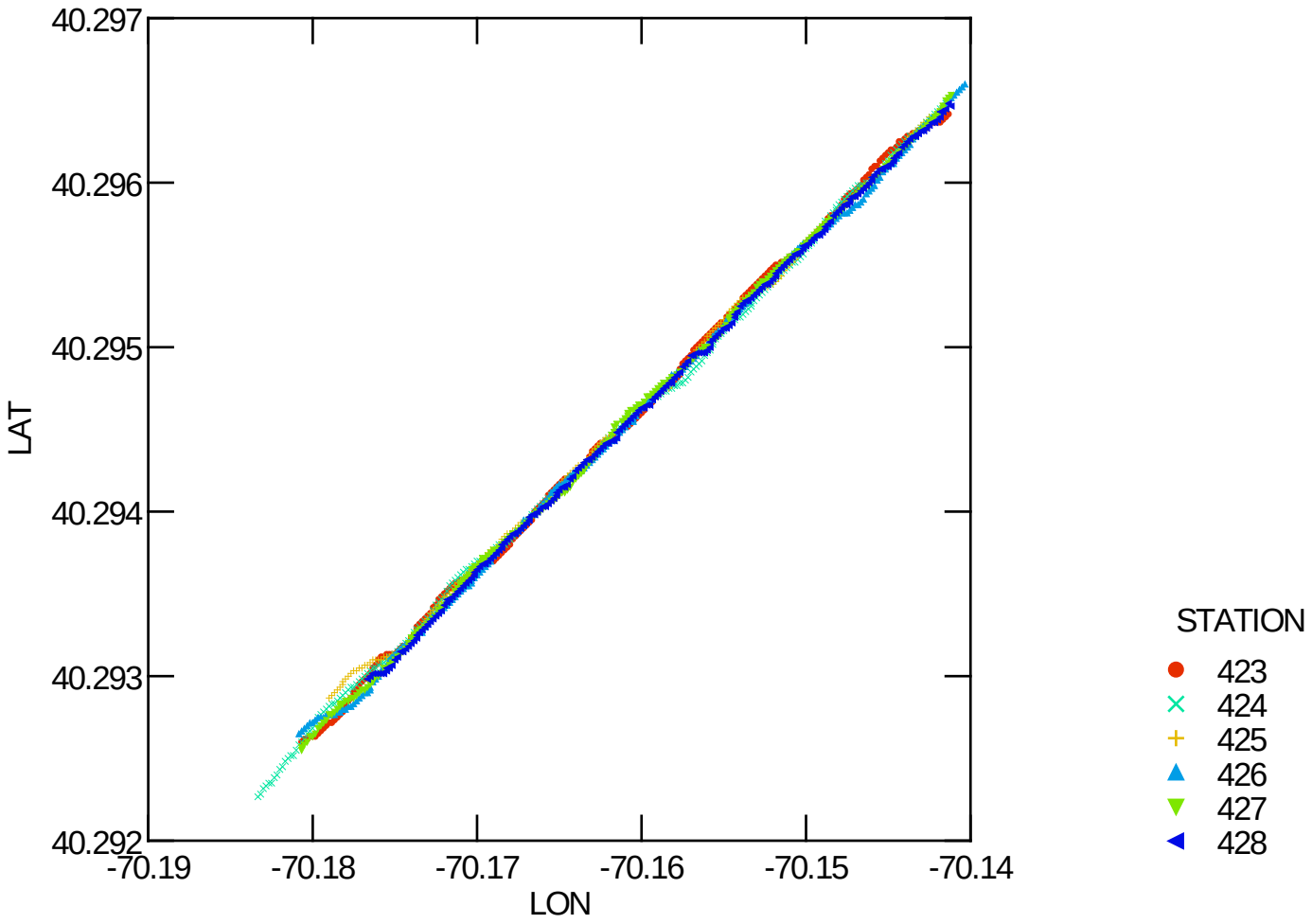

Experiment #5
HITFRAC
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
TOW
HITNUM
1
2
3
4
5
6

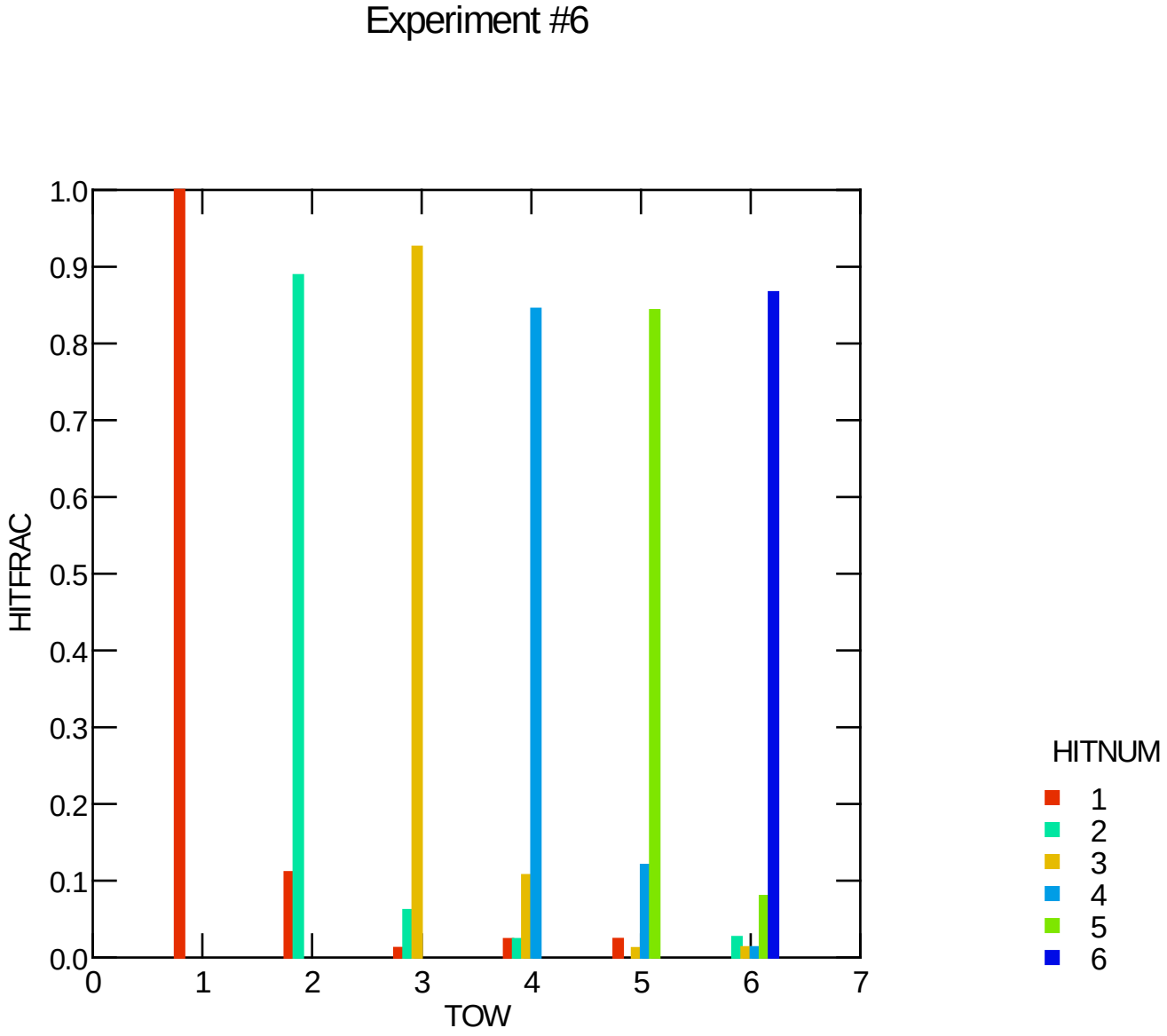
Experiment #5



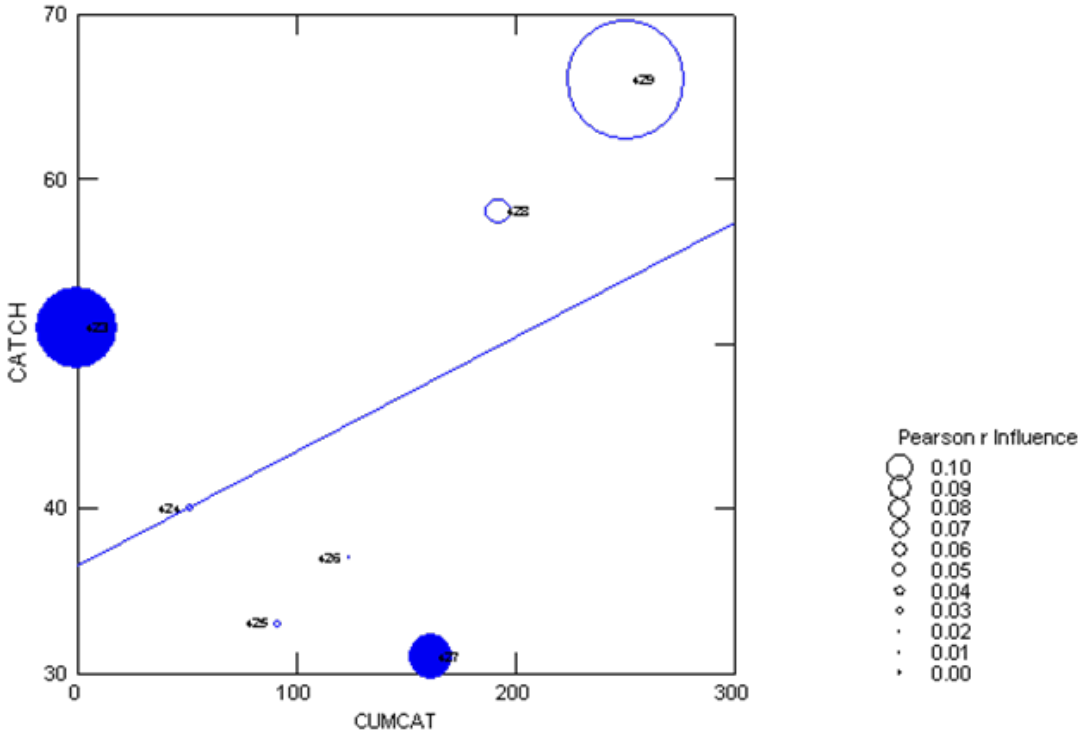
Tow	Area Swept	Efftv Area	C_5%	C_50%	C_pred	OBS	C_95%
CI 1	682690.0	261011.9	30.	46.	45.	46.	65.
CI 2	724422.0	200259.1	23.	35.	35.	26.	51.
CI 3	655756.0	130296.5	14.	23.	23.	26.	35.
CI 4	615450.0	90080.8	9.	16.	16.	17.	25.
CI 5	658380.0	84510.7	9.	15.	15.	23.	24.
CI 6	667457.0	58153.5	6.	11.	10.	4.	18.


ROW
COL
Count
ROW
COL
TOW
7
6
5
4
3
2
1
0

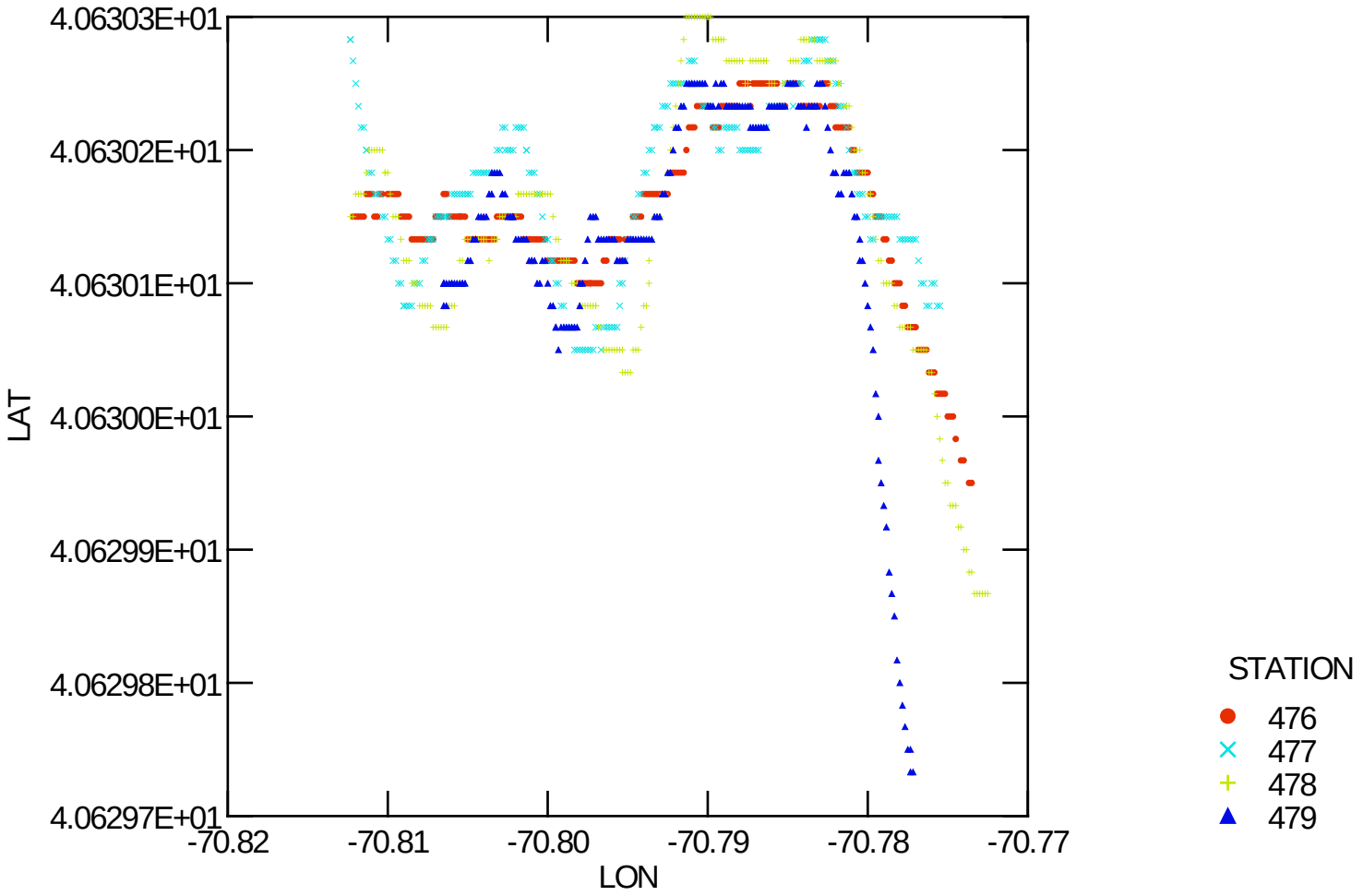

experiment #6
40.297
40.296
40.295
40.294
40.293
40.292
LAT
-70.19
-70.18
-70.17
-70.16
-70.15
-70.14
LON
STATION
423
424
425
426
427
428

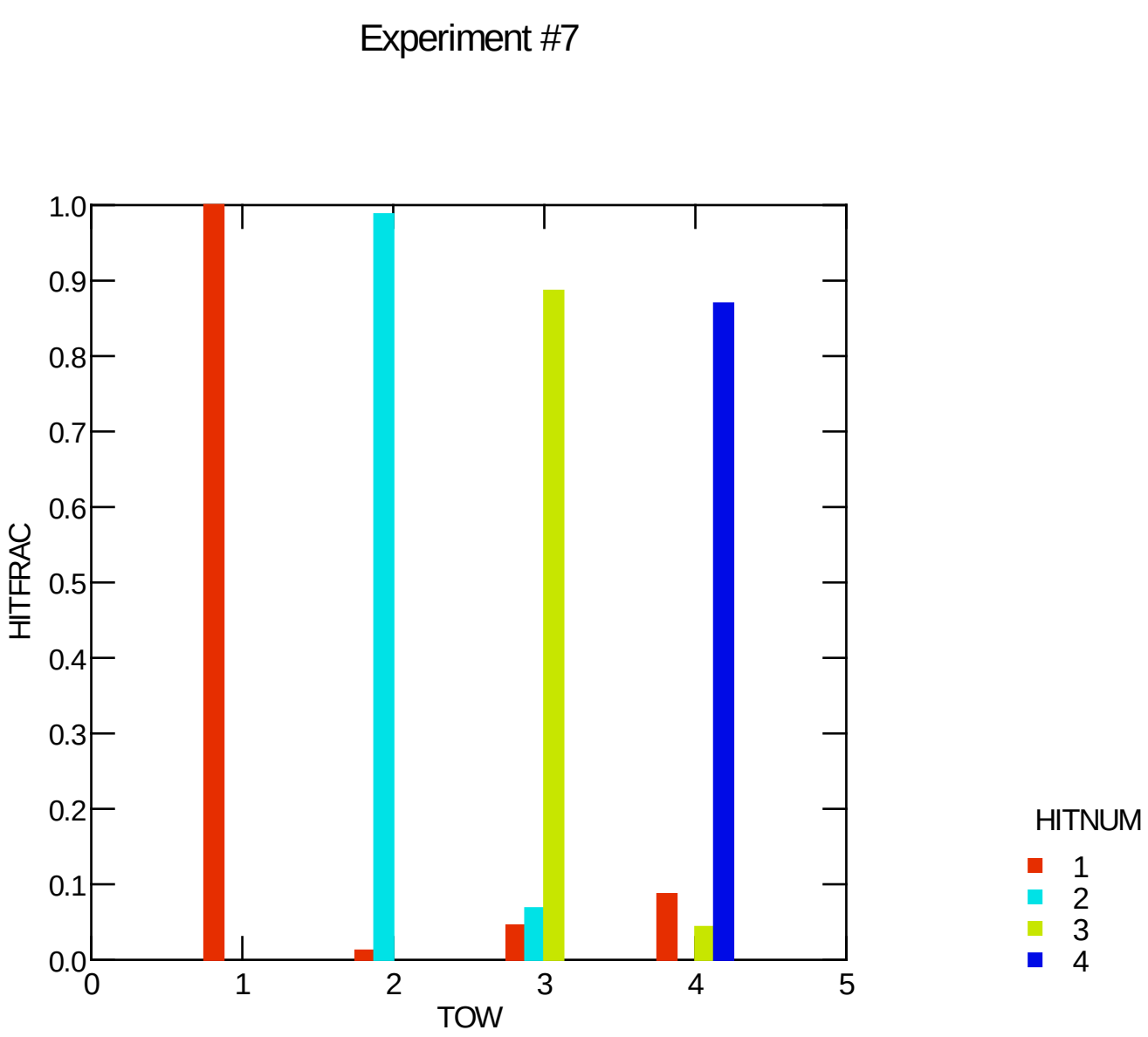

Experiment #6
HITFRAC
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
TOW
HITNUM
1
2
3
4
5
6

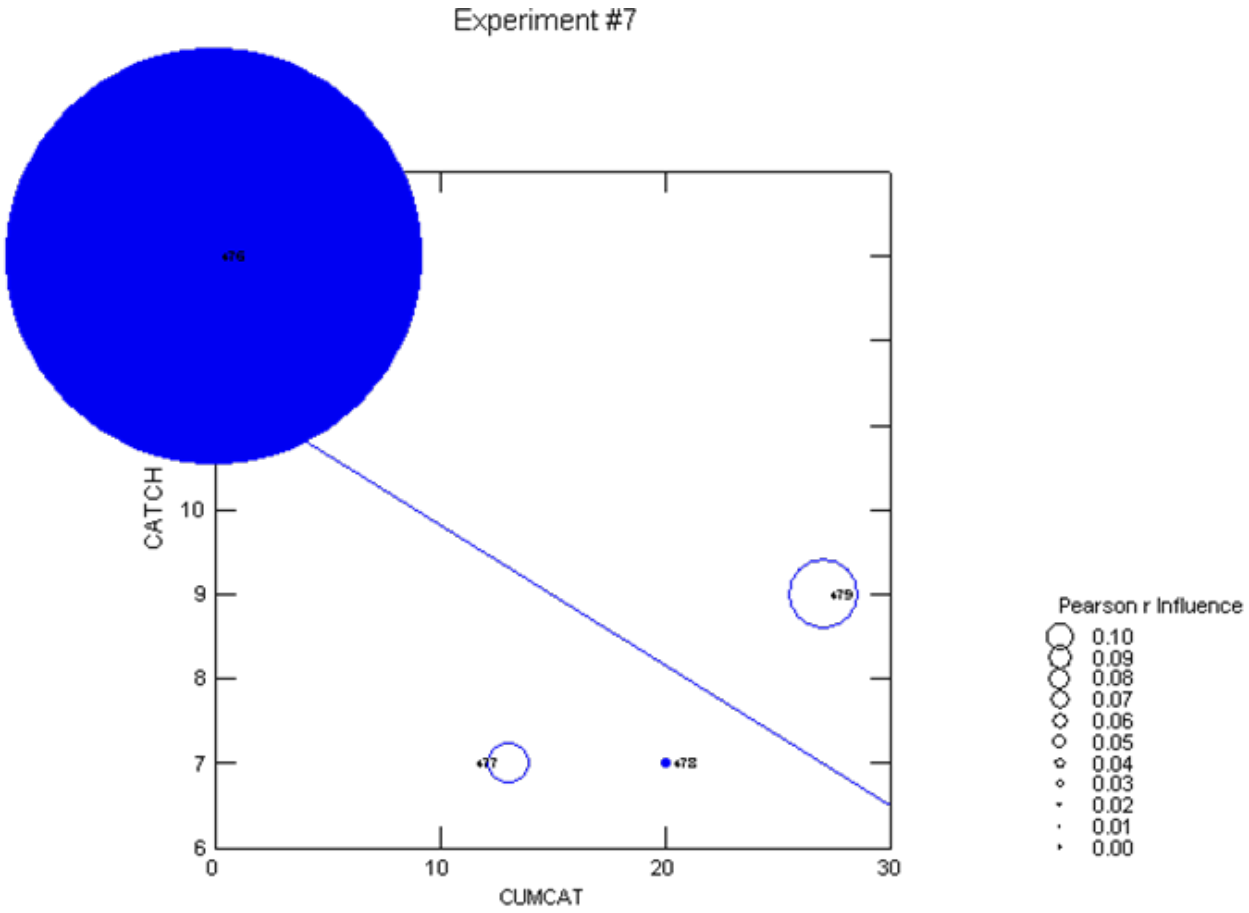
Experiment #6



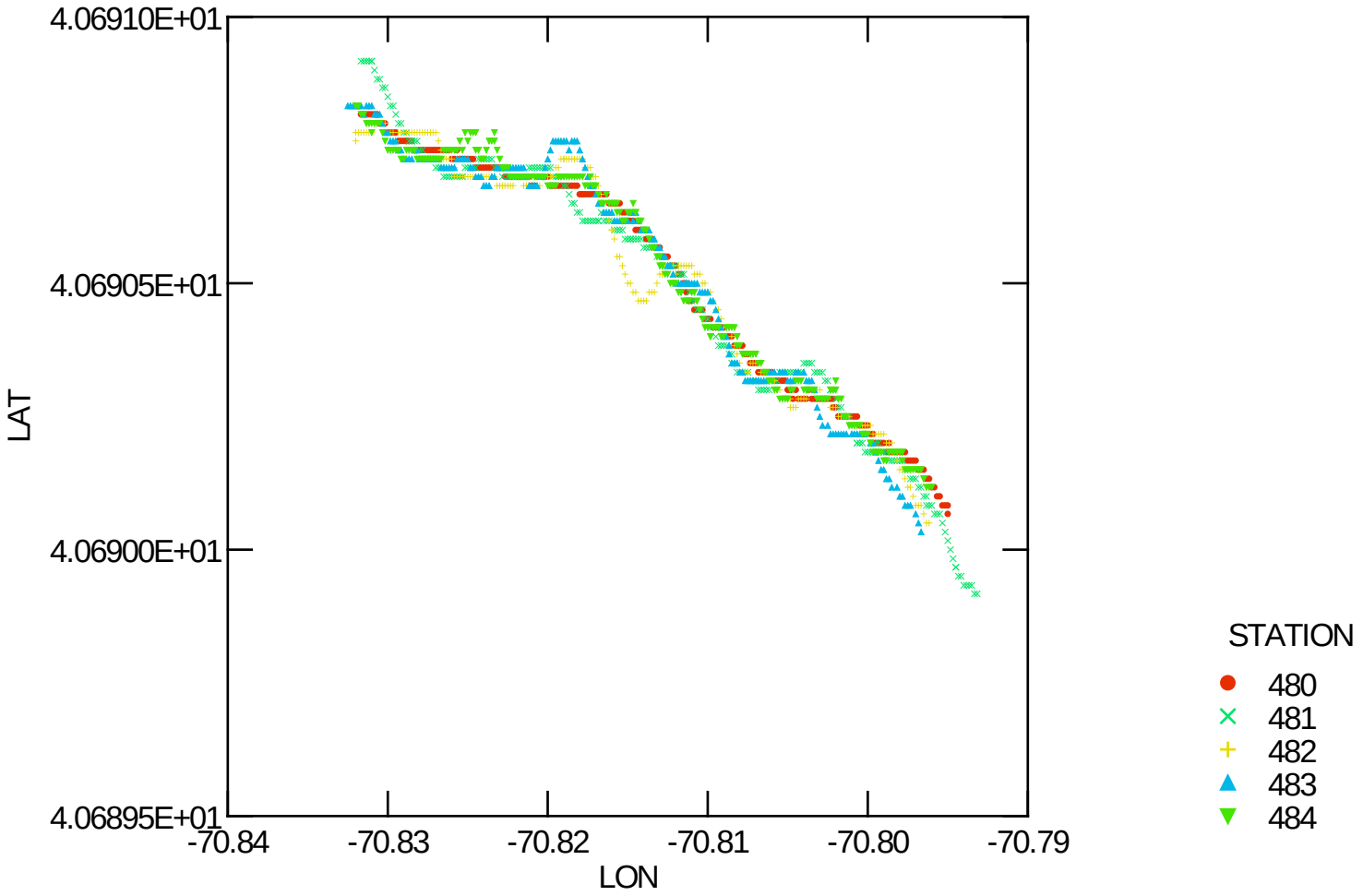
	Tow	Area Swept	Efftv Area	C_5%	C_50%	C_pred	OBS	C_95%
CI	1	1149328.0	57466.9	28.	47.	47.	51.	71.
CI	2	1228838.0	59257.8	29.	48.	48.	40.	73.
CI	3	1101188.0	50926.9	25.	41.	41.	33.	63.
CI	4	1180802.0	52736.3	26.	43.	43.	37.	65.
CI	5	1158107.0	49689.5	24.	40.	40.	31.	62.
CI	6	1031628.0	42518.2	20.	35.	34.	58.	53.


experiment #7
4.06303E+01
4.06302E+01
4.06301E+01
4.06300E+01
4.06299E+01
4.06298E+01
4.06297E+01
LAT
-70.82
-70.81
-70.80
-70.79
-70.78
-70.77
LON
STATION
476
477
478
479

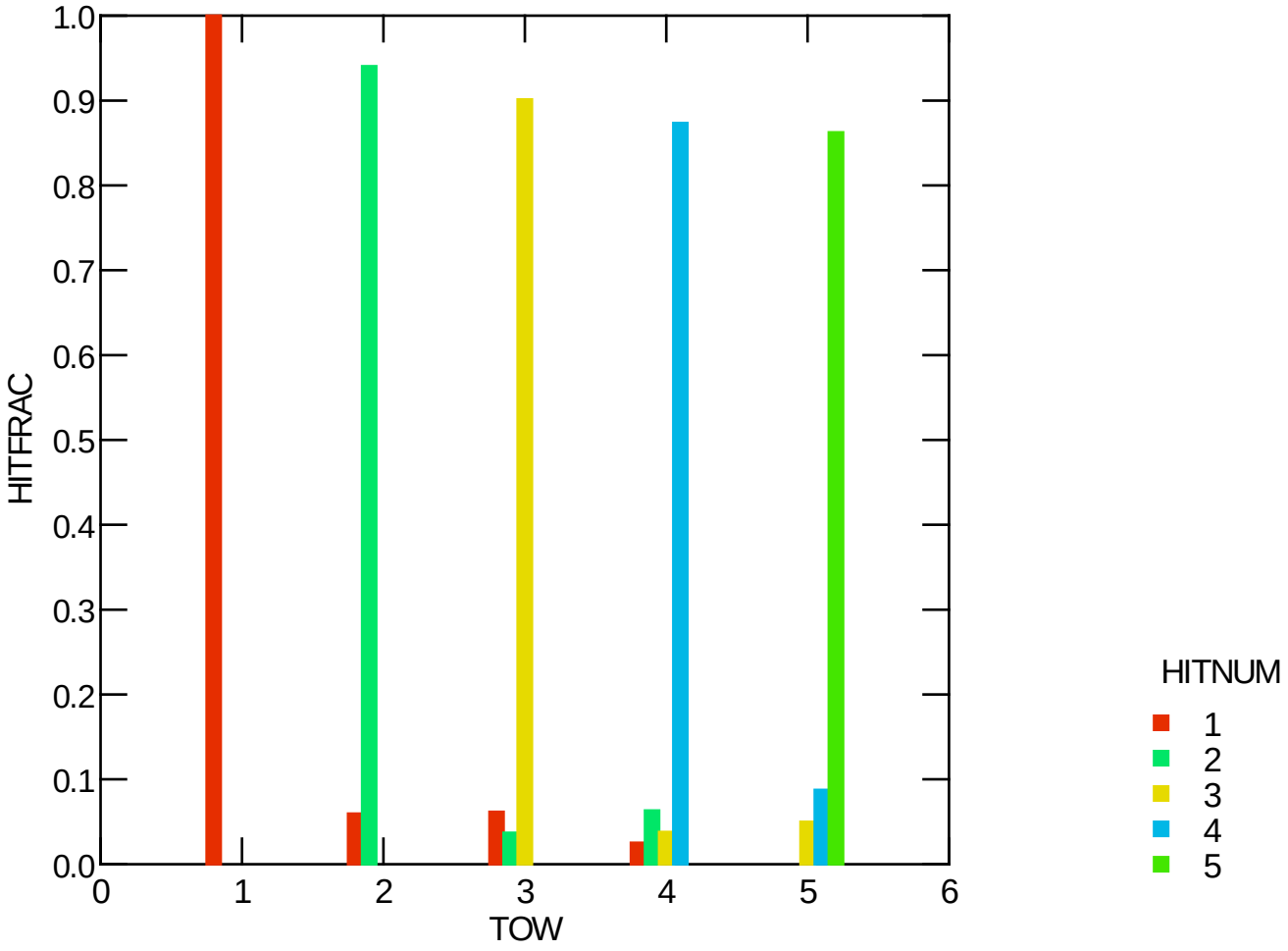

Experiment #7
HITFRAC
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
TOW
HITNUM
1
2
3
4

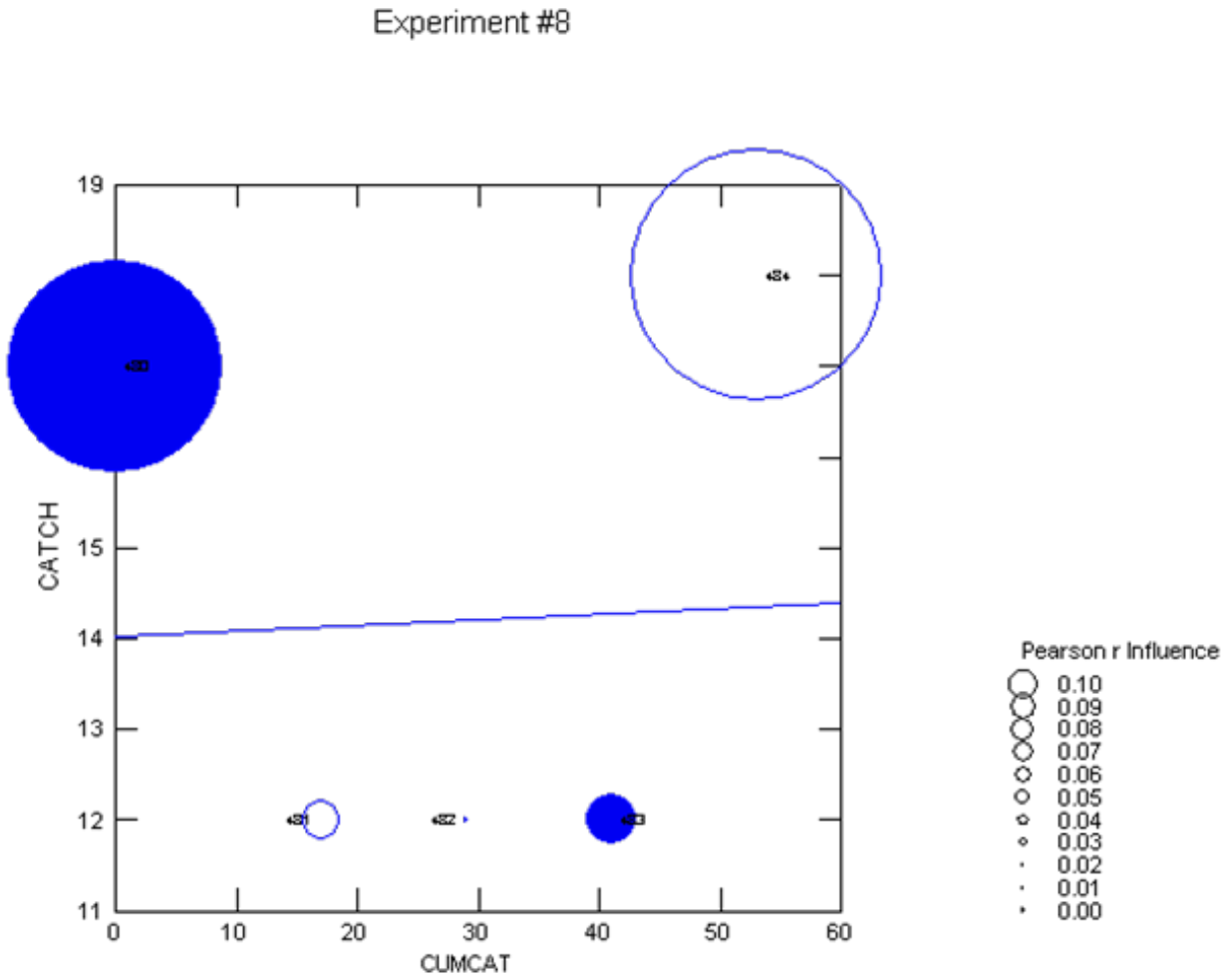


	Tow	Area Swept	Efftv Area	C_5%	C_50%	C_pred	OBS	C_95%
CI	1	928448.0	107975.1	7.	12.	11.	13.	17.
CI	2	894369.0	94451.2	6.	10.	9.	7.	16.
CI	3	966692.0	94029.9	6.	10.	9.	7.	16.
CI	4	719680.0	64567.9	4.	7.	6.	9.	12.


experiment #8
4.06910E+01
4.06905E+01
4.06900E+01
4.06895E+01
LAT
-70.84
-70.83
-70.82
-70.81
-70.80
-70.79
LON
STATION
480
481
482
483
484

Experiment #8





Tow		Area Swept	Efftv Area	C_5%	C_50%	C_pred	OBS	C_95%
CI	1	926410.0	46320.6	10.	16.	15.	17.	23.
CI	2	969823.0	46667.1	10.	16.	16.	12.	23.
CI	3	899735.0	41741.6	9.	15.	14.	12.	21.
CI	4	891727.0	39852.1	9.	14.	13.	12.	21.
CI	5	904538.0	38716.8	8.	14.	13.	18.	20.